SECTION 3. COMPUTER SCIENCE

3.1 Information technology of support management decision making of hierarchical organizational structure choice under uncertainty

Among the most important tasks of organizational structures (OS) management are closely related tasks of management of their structure and composition [153]. Such tasks are usually reduced to finding and forming optimal management hierarchies. The selection of the optimal hierarchy H_{opt}, from a set of hierarchies Ω such that $H_{opt} \in \Omega$, is carried out according to a certain set of criteria (for example, the cost of maintaining the staff of the organization).

Then the optimal hierarchy will be the one that meets the condition:

$$H_{opt} \in Arg \min_{H \in \Omega} C(H), \tag{1}$$

where C is a cost.

In [153] it is argued that for any hierarchy $H_1 \in \Omega$ there is a hierarchy $H_2 \in \Omega$ whose costs do not exceed the costs of maintaining the hierarchy H_1, i.e. $C(H_2) \leq C(H_1)$, and which satisfies the conditions:

1. no duplication, in which two managers manage the same group of performers;

2. the existence of only one manager (top manager), who has no superiors, and only he/she is subordinate to all other managers and executors of the hierarchy;

3. supports the so-called "normal" form of organization: the manager M_0 does not directly manage the subordinates of the manager M_1, who is subordinate to him/her.

The searching for optimal OS is carried out by searching the set of allowable hierarchies, which is quite a difficult task and requires large computing resources. The problem of optimization of hierarchical OS can be solved by using graph-dynamic modeling methods [154-158].

The theory of graph-dynamic systems [159, 160] is based on a set of operations that allow to obtain a sequence of hierarchical graphs. In Fig. 1 an example of graphical representation of graph-dynamic modeling of norms of controllability by hierarchical structures is given. The norm of controllability r characterizes the maximum number of direct subordinates controlled by one manager within $1 \leq r \leq n$ (n is the number of structural elements) [161]. The number of structures m generated in this way can be quite large, for example $m > 10$. Which can create difficulties with their ranking by preference.

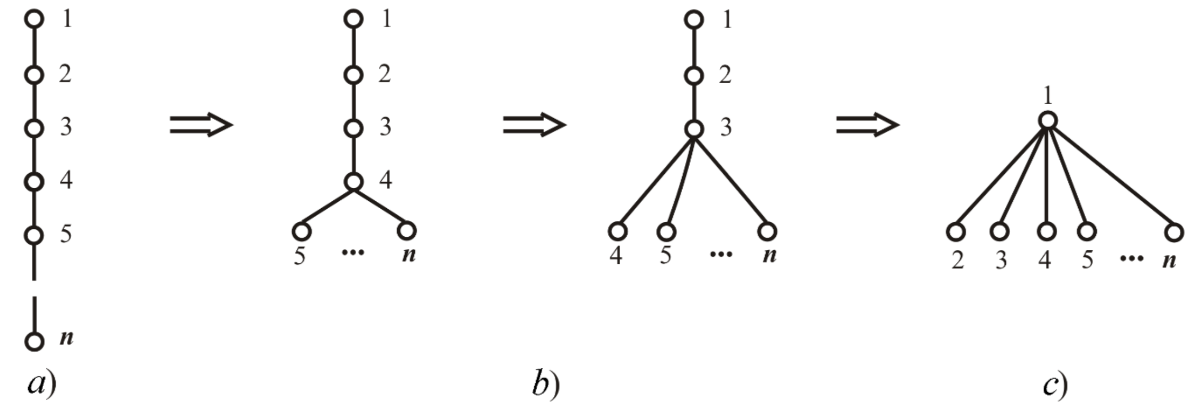

Figure 1. Graphical representation of graph-dynamic modeling of norms of controllability by hierarchical structures:
a) a chain graph (*r*-min); *b*) an intermediate hierarchies; *c*) a fan graph (*r*-max).

Currently, a fairly large class of multicriteria decision analysis methods has been formed, such as expert evaluation (scoring method, analytic hierarchy process, ELECTRE method, etc.), which are usually used at the initial stage of analysis, and various simulation methods for adoption the final solution.

It should be noted that the above methods of the first group do not take into account the Edgeworth-Pareto principle, according to which the best solution should be chosen among the Pareto-optimal solutions [162]. However, quite often the Pareto set can consist of a number of alternative solutions, which complicates the choice of one optimal solution and leads to the problem of narrowing (compressing) such a set.

Currently, a number of approaches have been proposed to solve this problem [162-164, etc.], among which the most developed is the theory of the importance of criteria [164] and axiomatic approach, which formulates a number of axioms that impose certain requirements on behavioral style of decision maker in terms of its advantages in the process of choosing alternative solutions. In general, both approaches can be implemented by providing additional information ("quanta" of information) of a quantitative or qualitative nature regarding the criteria or benefits of decision maker. According to [163], most of the existing methods aimed at narrowing the Pareto set cannot be considered strictly justified, as they are based on certain subjective heuristic considerations and/or use information that is extremely difficult to detect from decision maker. This paves the way for finding additional approaches to solving the problem of finding optimal solutions on the formed Pareto set.

Modeling of organizational structures by graph-dynamic methods

In [165, 166] has been defined the concept of "graph-dynamic system" and proposed one of the possible ways to describe it is the "Π-functions" language. A graph-dynamic system is a dynamic system in which the object of change is its structure, which is described by a connection graph between individual elements. Let us to considering only such systems in which the connection graph is a tree.

Let there be a hierarchical numbered tree graph (Fig. 2(a)), where the number n of each vertex is strictly greater than the number of the "senior" vertex with the number m, from which the vertex n is adjacent, and the "root" of the tree is conditionally considered adjacent from the fictitious vertex with number 0. Associate each vertex n with the number $\varphi(n)=m$. For example, for the graph in Fig. 1, and we get the following sequence:

$$\varphi(1)=0;\ \varphi(2)=\varphi(3)=\varphi(4)=1;\ \varphi(5)=2;, \qquad (2)$$
$$\varphi(6)=\varphi(7)=\varphi(8)=\varphi(9)=3;\ \varphi(10)=4;\ ...$$

Then each hierarchical tree-like graph can be associated with a single-valued integer lattice function $\varphi(n)$ defined on the set of all non-negative numbers: 1,2,3,…, $N_{\max}$, where $N_{\max}$ is the maximum number of a vertex in the graph. The function $\varphi(n)$, which reflects the structure of subordination in the graph, is called the subordination function, or the Π-function. Graphs of all possible Π-functions, taking into account restrictions on the numbering of vertices, lie inside the region $0\le\varphi(n)\le n-1$ of the plane $\{n, \varphi(n)\}$. The Fig. 2(b) shows the Π-function of the graph in Fig. 2(a).

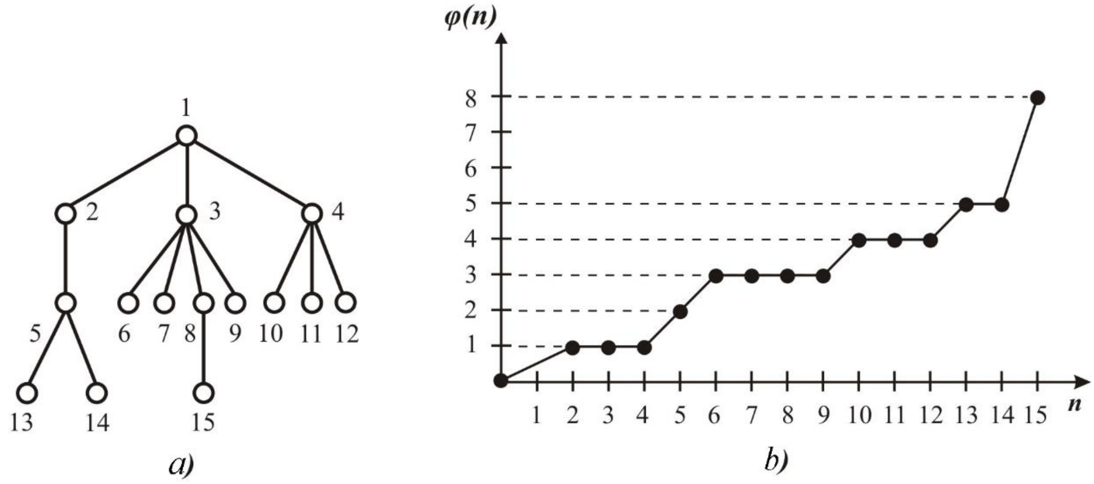

Figure 2. Tree-like graph with its Π-function

The integer Π -function φ(*n*) certainly satisfies the following restrictions:

1. φ(*n*)<*n*, i.e. the number of any vertex is always greater than the number of the vertex to which it is "subordinate".

2. φ(*n*) is defined on all integers *n* from 1 to some *N*.

3. φ(*n*)=0 means that the vertex of the graph with number *n* is not subordinate to any vertex, i.e. this constraint characterizes the procedure for the appearance of the root (root vertex) of a tree.

These procedures make it possible to transform hierarchical OS when solving optimization problems. Let's consider a number of examples illustrating the possibilities of graph-dynamic methods.

3.1.1 Modeling the procedure for increasing the number of subordinates in OS

Consider a situation where it is necessary to model the procedure for increasing the number of subordinates in a hierarchy as moving from higher to lower levels. Such a task can be considered as a redistribution of the norm of controllability among managers. The procedure can be implemented through the following operation:

$$\psi(n) = [\sqrt{\zeta(n)}],\ \zeta(n) = [n \cdot (\phi(n)+1)/2]. \qquad (3)$$

Figures 3(a) and 3(b) show the original binary tree and the transformed graph, respectively.

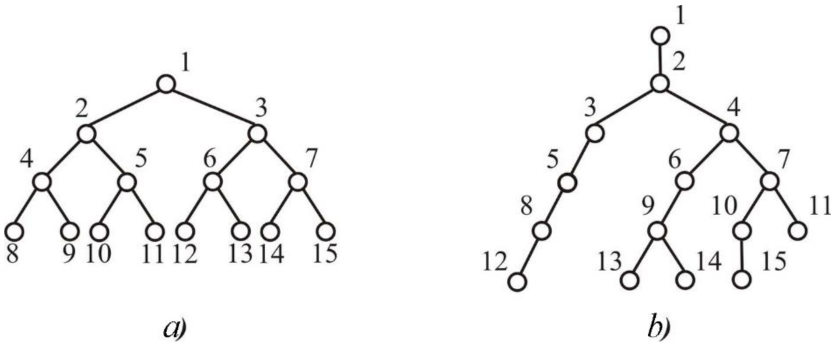

Figure 3. *a*) the original binary tree; *b*) the transformed graph

Let us write out the expressions for such a transformation:

$n=1$: $\zeta(n)=[1\cdot(0+1)/2]=1/2$; $\psi(n)=[\sqrt{\zeta(n)}]=[\sqrt{1/2}]=[0,7]=0$;

the first vertex is a root;

$n=2$: $\zeta(n)=[2\cdot(1+1)/2]=2$; $\psi(n)=[\sqrt{2}]=[1,4]=1\Rightarrow 2\prec 1$;

$n=3$: $\zeta(n)=[3\cdot(2+1)/2]=4,5$; $\psi(n)=[\sqrt{4,5}]=[2,12]=2\Rightarrow 3\prec 2$;

$n=4$: $\zeta(n)=[4\cdot(3+1)/2]=8$; $\psi(n)=[\sqrt{8}]=[2,88]=2\Rightarrow 4\prec 2$;

$n=5$: $\zeta(n)=[5\cdot(4+1)/2]=12,5$; $\psi(n)=[\sqrt{12,5}]=[3,53]=3\Rightarrow 5\prec 3$;

$n=6$: $\zeta(n)=[6\cdot(5+1)/2]=18$; $\psi(n)=[\sqrt{18}]=[4,2]=4\Rightarrow 6\prec 4$;

$n=7$: $\zeta(n)=[7\cdot(6+1)/2]=24,5$; $\psi(n)=[\sqrt{24,5}]=[4,94]=4\Rightarrow 7\prec 4$;

$n=8$: $\zeta(n)=[8\cdot(7+1)/2]=32$; $\psi(n)=[\sqrt{32}]=[5,65]=5\Rightarrow 8\prec 5$;

………………………………………………………………………………

$n=15$: $\zeta(n)=[15\cdot(14+1)/2]=112,5$; $\psi(n)=[\sqrt{112,5}]=[10,6]=10\Rightarrow 15\prec 10$.

The consideration of the transformed graph shows that the desired redistribution of the norm of controllability has taken place. For example, if in the original graph vertices 8 and 9 were subordinate to vertex 4, then in the transformed graph vertices 6, 7, 9, 10, 11, 13, 14, 15 are already subordinate to it.

3.1.2 Modeling the procedure for the hierarchy integration (aggregation)

Figure 4 shows three initial structures and their П-functions ($\phi_1(n)$, $\phi_2(n)$, $\phi_3(n)$), from which it is necessary to create one structure.

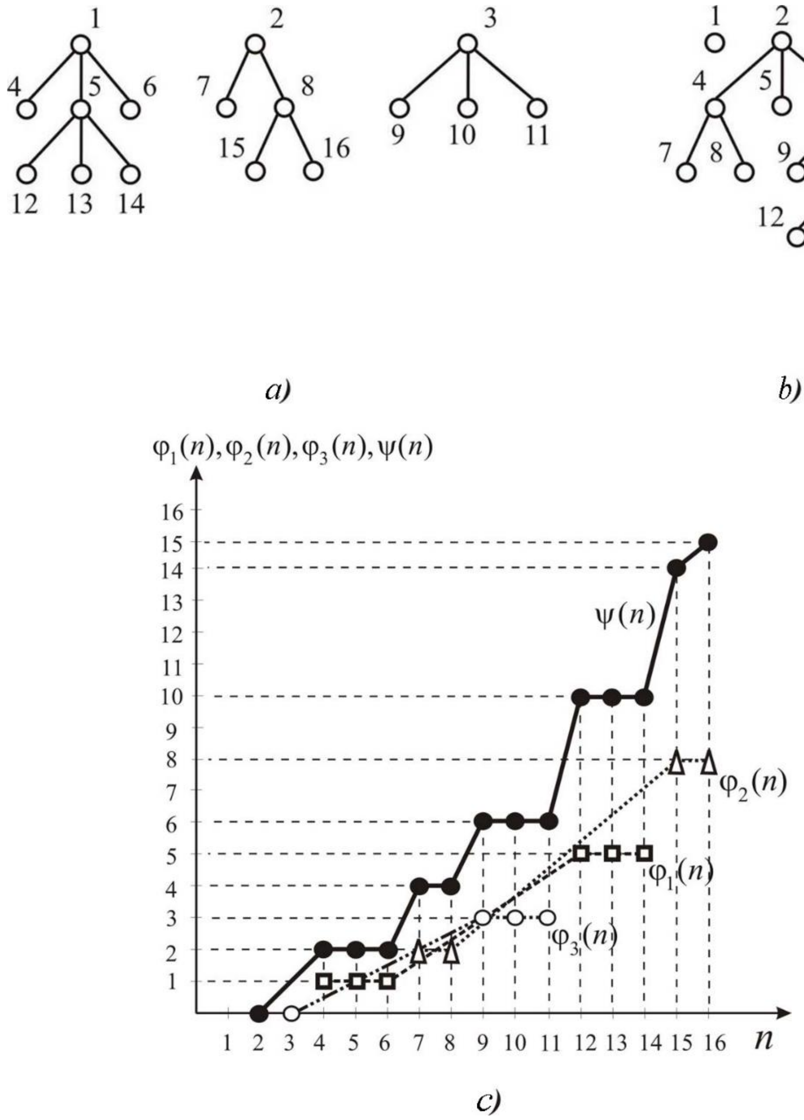

Figure 4. Graphical representation of the structure aggregation procedure

a) initial structures; *b*) aggregation structure; *c*) П-functions of structures

The "aggregation" operation was used for this transformation:

$$\psi(n) = k \cdot \phi(n)\text{, } k{=}2{,}3{,}\ldots. \tag{4}$$

The necessary transformations are presented below:

$n=1: \psi(n) = 2 \cdot \phi(0) = 0 \Rightarrow$ the first vertex is a root;

$n=2: \psi(n) = 2 \cdot \phi(0) = 0 \Rightarrow$ the second vertex is a root;

$n=3: \psi(n) = 2 \cdot \phi(0) = 0 \Rightarrow$ the third vertex is a root;

$n=8: \psi(n) = 2 \cdot \phi(2) = 4 \Rightarrow 8 \prec 4$;

$n=9: \psi(n) = 2 \cdot \phi(3) = 6 \Rightarrow 9 \prec 6$;

$n=10: \psi(n) = 2 \cdot \phi(3) = 6 \Rightarrow 10 \prec 6$;

$n=11: \psi(n) = 2 \cdot \phi(3) = 6 \Rightarrow 11 \prec 6$;

$n=12: \psi(n) = 2 \cdot \phi(5) = 10 \Rightarrow 12 \prec 10$;

$n=13: \psi(n) = 2 \cdot \phi(5) = 10 \Rightarrow 13 \prec 10$;

$n=4:\psi(n)=2\cdot\phi(1)=2\Rightarrow 4\prec 2;$

$n=5:\psi(n)=2\cdot\phi(1)=2\Rightarrow 5\prec 2;$

$n=6:\psi(n)=2\cdot\phi(1)=2\Rightarrow 6\prec 2;$

$n=7:\psi(n)=2\cdot\phi(2)=4\Rightarrow 7\prec 4;$

$n=14:\psi(n)=2\cdot\phi(5)=10\Rightarrow 14\prec 10;$

$n=15:\psi(n)=2\cdot\phi(8)=16;$

$n=16:\psi(n)=2\cdot\phi(8)=16.$

In the last two expressions for n=15 and n=16, the resulting functions ψ(n) violate the conditions for its existence, i.e. $\psi(n)>n$. Therefore, proceeding from [165, 166], we use the procedure for its extension: $\psi(n)=(n-1)$.

Then the expressions for n=15 and n=16 can be represented as follows:

$$n=15:\psi(n)=2\cdot\phi(8)=16\Rightarrow(n-1=15-1=14);15\prec 14;$$

$$n=16:\psi(n)=2\cdot\phi(8)=16\Rightarrow(n-1=16-1=15);15\prec 15.$$

Figure 4(b) shows the transformed graph that defines the aggregated OS.

3.1.3 Modeling the procedure for the aggregation of hierarchy with a reduction its components

To implement such a procedure, the "aggregation with reduction" operation was used:

$$\psi(n)=\varphi(k\cdot n),\ k=2.3,\ldots. \tag{5}$$

This operation reduces the number of tree nodes while simultaneously increasing the number of hierarchy levels. Figure 5 shows five initial structures and their subordination functions.

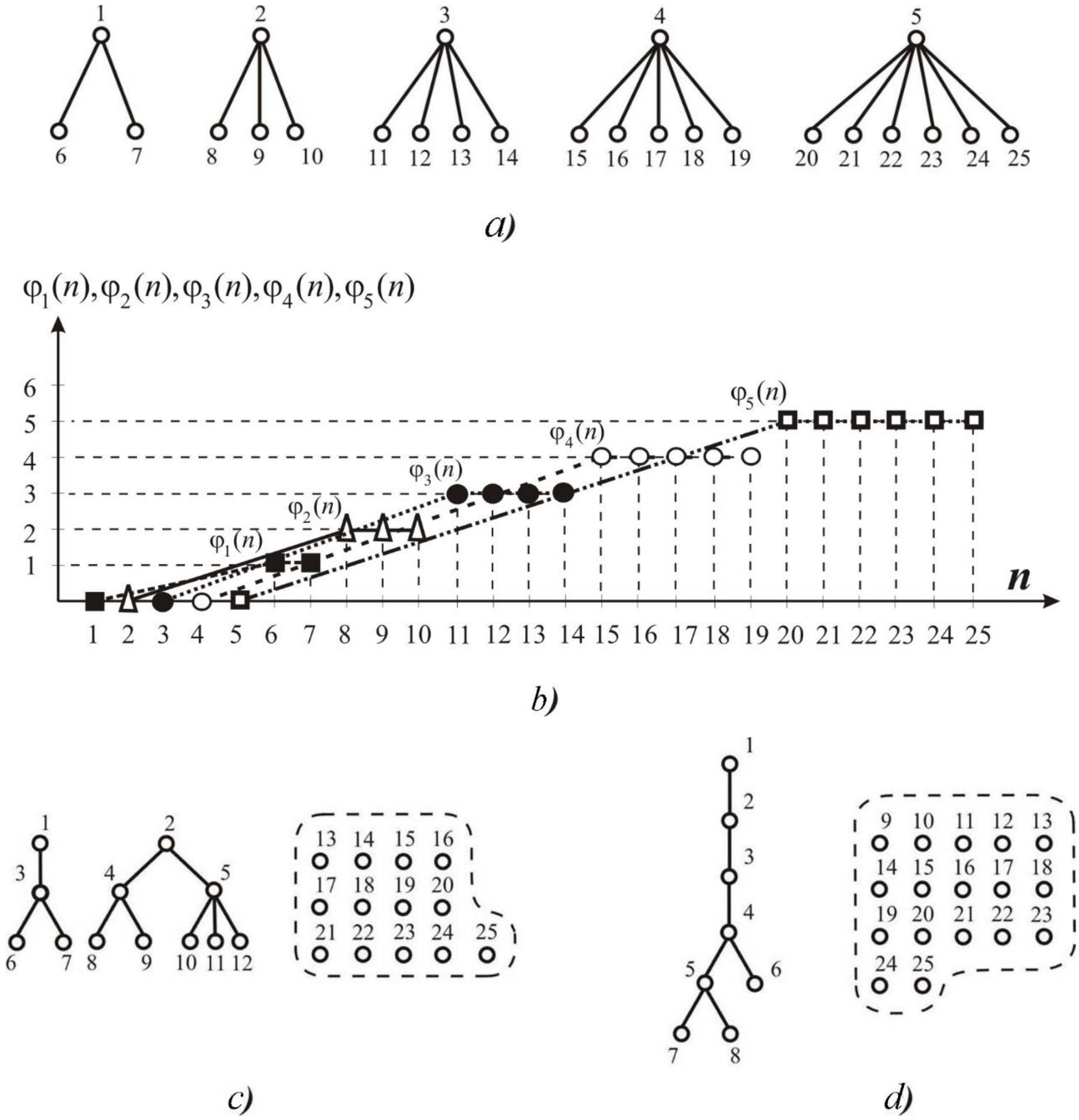

Figure 5. Graphical representation of the structure aggregation procedure: *a*) initial structures; *b*) Π -functions of initial structures; *c*) an aggregated structure with a reduction its components for *k*=2; *d*) an aggregated structure with a reduction its components for *k*=3

Let's perform transformations of initial structures for *k*=2 and *k*=3.

For *k*=2:

$n=1: \psi(n)=\phi(2\cdot 1)=0 \Rightarrow$ the first vertex is a root;

$n=2: \psi(n)=\phi(2\cdot 2)=0 \Rightarrow$ the second vertex is a root;

$n=7: \psi(n)=\phi(2\cdot 7)=3 \Rightarrow 7 \prec 3;$

$n=8: \psi(n)=\phi(2\cdot 8)=4 \Rightarrow 8 \prec 4;$

$n=9: \psi(n)=\phi(2\cdot 9)=4 \Rightarrow 9 \prec 4;$

$n=10: \psi(n)=\phi(2\cdot 10)=5 \Rightarrow 10 \prec 5;$

$n=3:\psi(n)=\phi(2\cdot 3)=1\Rightarrow 3\prec 1;$

$n=4:\psi(n)=\phi(2\cdot 4)=2\Rightarrow 4\prec 2;$

$n=5:\psi(n)=\phi(2\cdot 5)=2\Rightarrow 5\prec 2;$

$n=6:\psi(n)=\phi(2\cdot 6)=3\Rightarrow 6\prec 3;$

$n=11:\psi(n)=\phi(2\cdot 11)=5\Rightarrow 11\prec 5;$

$n=12:\psi(n)=\phi(2\cdot 12)=5\Rightarrow 12\prec 5;$

$n=13:\psi(n)=\phi(2\cdot 13)$ – does not exist.

For n=14÷25 the $\psi(n)$ function does not exist.

For k=3:

$n=1:\psi(n)=\phi(3\cdot 1)=0\Rightarrow$ the first vertex is a root;

$n=2:\psi(n)=\phi(3\cdot 2)=1\Rightarrow 2\prec 1;$

$n=3:\psi(n)=\phi(3\cdot 3)=2\Rightarrow 3\prec 2;$

$n=4:\psi(n)=\phi(3\cdot 4)=3\Rightarrow 4\prec 3;$

$n=5:\psi(n)=\phi(3\cdot 5)=4\Rightarrow 5\prec 4;$

$n=6:\psi(n)=\phi(3\cdot 6)=4\Rightarrow 6\prec 4;$

$n=7:\psi(n)=\phi(3\cdot 7)=5\Rightarrow 7\prec 5;$

$n=8:\psi(n)=\phi(3\cdot 8)=5\Rightarrow 8\prec 5;$

$n=9:\psi(n)=\phi(3\cdot 9)$ – does not exist.

For n=10÷25 the $\psi(n)$ function does not exist.

The resulting graphs are shown in Figures 5(c) and 5(d). The Figure 5(c) illustrates two trees (instead of five), as well as the presence of disconnected ("scattered") vertices 13÷25. In the second case (for k=3), one structure was obtained, and the number of disconnected vertices increased (from 9 to 25). The appearance of these vertices can be interpreted as a reduction in the composition of the organization.

Information technology of optimal OS selection

The paper proposes a methodology of synthesis the information technology of qualitative modeling of the problem of choosing the optimal OS, which is based on the integrated use of procedure for the Pareto set construction and the theory of evidence methods [167, 168].

For example, based on the method of graph-dynamic modeling, a number of variants of the OS of the enterprise as a whole, or its individual units $\mathrm{A}=\{A_i \mid i=\overline{1,n}\}$, were synthesized. It is necessary to select the most optimal OS option.

Let's consider the main stages of the proposed methodology for selecting the optimal OS of the enterprise, Fig. 6.

Stage 1. Narrowing the set of acceptable OS options based on a set of technical and economic indicators of the organization.

Let's select a set of indicators $\mathrm{K} = \{K_j \mid j = \overline{1,m}\}$ (production, economic, organizational and technical parameters) that determine the state of the analyzed enterprise, where $K_j = \{k_l^{(j)} \mid l = \overline{1,t}\}$ is a complex indicator, each $k_l^{(j)}$ is a measurable indicator of the enterprise.

On the basis of $k_l^{(j)}$ within each complex indicator K_j the problem of a choice of an optimum variant of OS (or several variants if the range of admissible values of the corresponding $k_l^{(j)}$ indicator is established) is solved.

Then, $\forall K_j \in \mathrm{K}: \ \mathrm{B}_j = \cup B_l^{*(j)}, l = \overline{1,t}; \ B_l^{*(j)} = \underset{i}{opt}\, k_l^{(j)}(A_i), \ B_l^{*(j)} \subset \mathrm{A}, \left|B_l^{*(j)}\right| = \overline{1,n}.$

Restructuring options $C = \underset{j=\overline{1,m}}{\cap}\left(A \setminus \mathrm{B}_j\right)$ are excluded from further analysis. Then $\forall A_i \in \mathrm{A} \setminus C : < A_i, \omega_i >$ will be formed, where ω_i is the number of indicators $k_l^{(i)}$ for which A_i is the optimum.

If $\left|\mathrm{A} \setminus C\right| > 1$ so, a set of additional analysis criteria is formed for expert evaluation of OS options.

If $\left|\mathrm{A} \setminus C\right| = 0$ so, it is recommended to make changes to:

1. a set of criteria for evaluation of the set of OS options;
2. a range of allowable $k_l^{(j)}$ values for A_i evaluation;
3. a set of parameters for A_i synthesis.

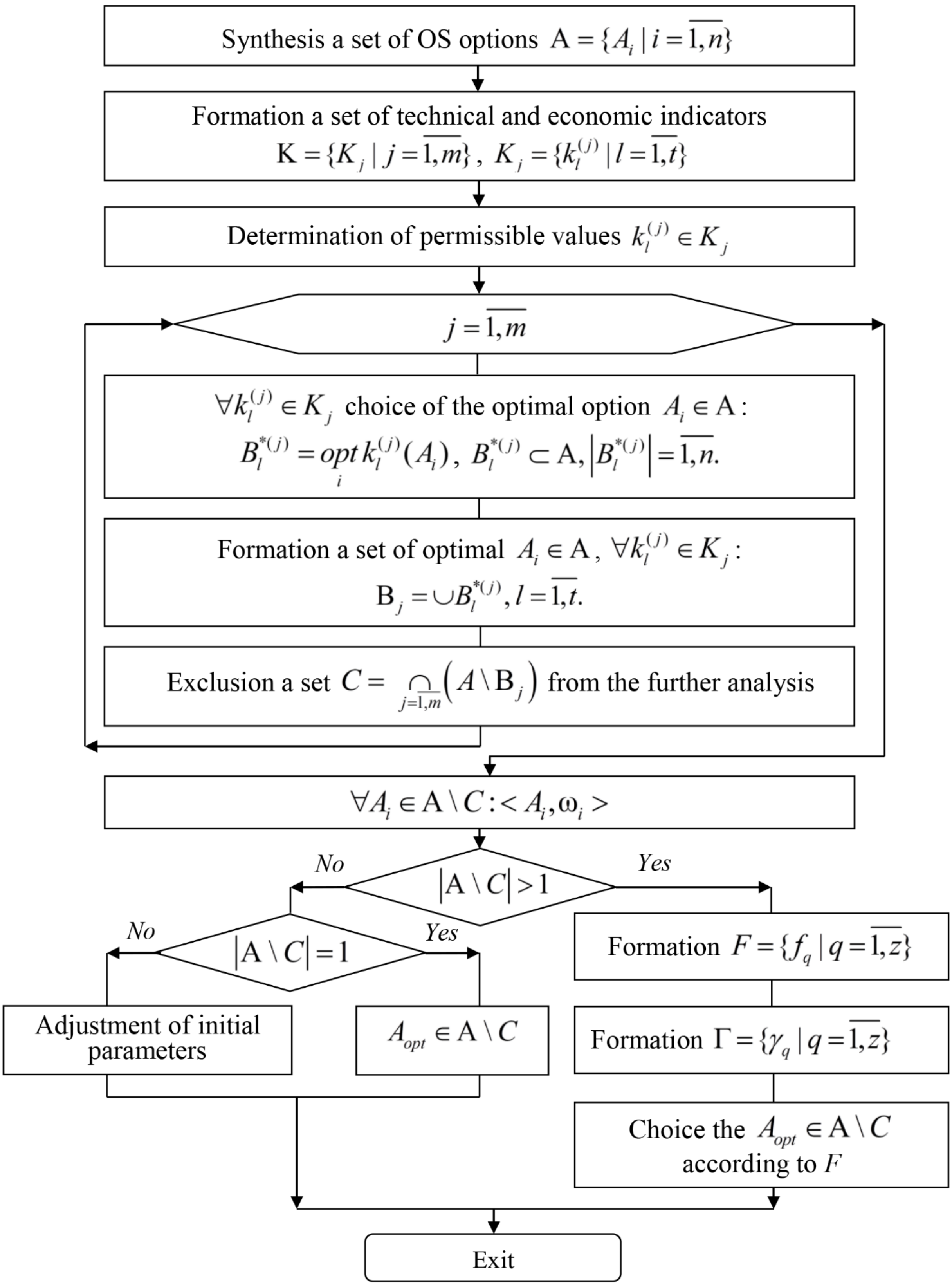

Figure 6. Algorithm for selection of the optimal OS of the enterprise

Stage 2. Selection of optimal OS options based on expert evaluation methods.

Let $\mathrm{X} = \mathrm{A} \setminus C = \{X_i \mid i = \overline{1, p}\}$ be a set of OS options ($p = |\mathrm{A} \setminus C|$), each element of which is evaluated by a set of local criteria $f_1, f_2, ..., f_z$, with some given coefficients of relative importance $\gamma_1, \gamma_2, ..., \gamma_z$.

Then, $F = \{f_q\}$ is an integral criterion ($q = \overline{1, z}$); $\Gamma = \{\gamma_q\}$ is a vector of importance of criteria $\{f_q\}$, $q = \overline{1, z}$.

The task is to choose the optimal solution *X**:

$$x^* = \underset{x \in \mathrm{X}}{opt}[F(x), \Gamma], \tag{6}$$

where *opt* is a some optimization operator.

The procedure for selection the optimal OS option according to the Pareto-optimality:

2.1. Selection of the set of Pareto-optimal solutions $P_f(\mathrm{X})$:

$$P_f(\mathrm{X}) = \left\{x^* \in \mathrm{X} \mid \overline{\exists} x \in \mathrm{X} : F(x) \geq F(x^*)\right\}. \tag{7}$$

2.2. Choosing the optimum among non-dominant solutions.

If $|P_f(\mathrm{X})| > 1$, then to select a single solution, or narrow the Pareto set, the following procedure is proposed:

1. Use the notation $\mathrm{A} = P_f(\mathrm{X})$, $\mathrm{A} = \{A_i \mid i = \overline{1, n}\}$, $n = |P_f(\mathrm{X})|$.

2. Form $2^{|\mathrm{A}|}$-1 subsets $B_i \subseteq \mathrm{A}$ based on the set A, that satisfy conditions [169-171]:

$$\begin{aligned} &1.\ B_i = \{\varnothing\}; \\ &2.\ B_i = \{A_j\},\ A_j \in \mathrm{A}; \\ &3.\ B_i = \{A_j \mid j = \overline{1, k}\},\ k < n,\ A_j \in \mathrm{A}. \\ &4.\ B_i = \mathrm{A} = \{A_j \mid j = \overline{1, n}\}. \end{aligned} \tag{8}$$

3. Determine the relationship $B_i \succeq \mathrm{A}$, $B_i \subseteq \mathrm{A}$; determine the appropriate degree of superiority $c_i \in C$ for all formed $B_i \subseteq \mathrm{A}$.

4. Form a truncated matrix of pairwise comparisons of selected subgroups of alternatives [172]:

$$\begin{pmatrix} 1 & 0 & \ldots & 0 & c_{1d} \\ 0 & 1 & \ldots & \vdots & c_{2d} \\ \ldots & \vdots & \ddots & 0 & \\ 0 & \ldots & 0 & 1 & c_{dd} \\ 1/c_{d1} & 1/c_{d2} & \ldots & 1/c_{dd} & 1 \end{pmatrix}, \tag{9}$$

where $c_{ij} = 1/c_{ji}$, $\forall\, i, j = \overline{1,d}$; c_{ij} is an expert preferences of formed by E_j subset $B_i \subseteq \mathrm{A}$; d is an amount of formed by E_j subsets $B_i \subseteq \mathrm{A}$.

5. Calculate the basic probability assignment $\mathrm{m} = \{m_i \mid i = \overline{1, 2^{|\mathrm{A}|} - 1|}\}$ for the all selected subsets in accordance with next expression [172]:

$$m_i = \frac{c_{id}}{\sum_{j=1}^{d} c_{jd} + \sqrt{d}}; \qquad m_{d+1} = \frac{\sqrt{d}}{\sum_{i=1}^{d} c_{id} + \sqrt{d}}. \tag{10}$$

6. $\forall E_j$, $j = \overline{1,t}$: aggregation of the obtained values $\mathrm{B} = \{B_i \mid i = \overline{1, 2^{|\mathrm{A}|} - 1}\}$ and $\mathrm{m} = \{m_i \mid i = \overline{1, 2^{|\mathrm{A}|} - 1|}\}$, provided that the evaluation was carried out by a group of experts $\mathrm{E} = \{E_j \mid j = \overline{1,t}\}$.

7. $\forall A_i \in \mathrm{A}$: formation of intervals $[Bel(\{A_i\}), Pl(\{A_i\})]$

Belief function $Bel : \Lambda \to [0,1]$:

$$Bel(B) = \sum_{X_j \subseteq B,\, X_j \in \Lambda} m(X_j). \tag{11}$$

Plausibility function $Pl : \Lambda \to [0,1]$:

$$Pl(B) = \sum_{X_j \cap B \neq \varnothing,\, X_j \in \Lambda} m(X_j), \tag{12}$$

where Λ denotes 2^{A}.

8. Choosing the optimal solution $A_{opt} \in \mathrm{A}$ by comparing the intervals $[Bel(\{A_i\}), Pl(\{A_i\})]$, $\forall i = \overline{1,n}$: $\exists A_{opt} = A_i$, provided that $\max_i [Bel(\{A_i\}), Pl(\{A_i\})]$.

Let us consider an example of the implementation of the proposed methodology. For example, on the basis of the method of graph-dynamic modeling was synthesized 20 variants of the OS of the enterprise as a whole, or its individual units $\mathrm{A} = \{A_i \mid i = \overline{1,20}\}$. The following complex indicators $\mathrm{K} = \{K_j \mid j = \overline{1,3}\}$ were used to evaluate each A_i option [173]:

K_1 – a group of indicators that express the results of the enterprise activity;

K_2 – a group of indicators that characterize the organization of the management process;

K_3 – a group of indicators that characterize the rationality of the OS and its technical and organizational level.

Let's define a set of measured indicators of activity of the enterprise on each defined above complex indicator [173]:

$K_1 = \{k_l^{(1)} \mid l = \overline{1,3}\}$ – saving capital investments; increase in production volumes; cost reduction.

$K_2 = \{k_l^{(1)} \mid l = \overline{1,3}\}$ – productivity; economy; reliability.

$K_3 = \{k_l^{(1)} \mid l = \overline{1,4}\}$ – duplication factor (coefficient); degree of centralization; controllability level coefficient; coefficient of territorial concentration.

According to the results of the analysis of the synthesized set of OS options $\mathrm{A} = \{A_i \mid i = \overline{1,20}\}$, the set A was narrowed to 6 options $\mathrm{A}^* = \{A_i \mid i = \overline{1,6}\}$, $\mathrm{A}^* \subseteq \mathrm{A}$, which are evaluated by the formed set $m = 8$ of local criteria ($f_1 \div f_8$), for example:

f_1 – flexibility of the control system;

f_2 – adaptability of the control system;

f_3 – the possibility of diversification without significant loss of control over activities;

f_4 – the degree of centralization of financial resources;

f_5 – independence of units;

f_6 – maneuverability of resources;

f_7 – efficiency of control;

f_8 – balance of rights and responsibilities of managers and executors.

Let us consider an example of constructing a Pareto set based on the proposed approach.

To evaluate alternatives for each criterion, a five-point scale has been used. The results are shown in Table 1.

Table 1.

Profiles of expert preferences

Alternatives (OS options)	Criteria							
	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
A_1	4	3	4	3	1	4	3	5
A_2	5	3	3	3	2	3	4	4
A_3	2	4	2	4	4	2	3	5
A_4	5	3	2	3	2	1	4	3
A_5	3	4	3	4	5	2	4	2
A_6	4	3	5	4	4	4	3	5

According to the Pareto set search algorithm, following pairwise comparisons has been done:

1. Compare in pairs A_1 and A_2; A_1 and A_3; A_1 and A_4; A_1 and A_5; A_1 and A_6 according to each criterion $f_1 \div f_8$.

$$A_1 \underset{K_1}{<} A_2;\ A_1 \underset{K_2}{=} A_2;\ A_1 \underset{K_3}{>} A_2;\ A_1 \underset{K_4}{=} A_2;\ A_1 \underset{K_5}{<} A_2;\ A_1 \underset{K_6}{>} A_2;\ A_1 \underset{K_7}{<} A_2;\ A_1 \underset{K_8}{>} A_2;$$

$$A_1 \underset{K_1}{>} A_3;\ A_1 \underset{K_2}{<} A_3;\ A_1 \underset{K_3}{>} A_3;\ A_1 \underset{K_4}{<} A_3;\ A_1 \underset{K_5}{<} A_3;\ A_1 \underset{K_6}{>} A_3;\ A_1 \underset{K_7}{=} A_3;\ A_1 \underset{K_8}{=} A_3;$$

$$A_1 \underset{K_1}{<} A_4;\ A_1 \underset{K_2}{=} A_4;\ A_1 \underset{K_3}{>} A_4;\ A_1 \underset{K_4}{=} A_4;\ A_1 \underset{K_5}{<} A_4;\ A_1 \underset{K_6}{>} A_4;\ A_1 \underset{K_7}{<} A_4;\ A_1 \underset{K_8}{>} A_4;$$

$$A_1 \underset{K_1}{>} A_5;\ A_1 \underset{K_2}{<} A_5;\ A_1 \underset{K_3}{>} A_5;\ A_1 \underset{K_4}{<} A_5;\ A_1 \underset{K_5}{<} A_5;\ A_1 \underset{K_6}{>} A_5;\ A_1 \underset{K_7}{<} A_5;\ A_1 \underset{K_8}{>} A_5;$$

$$A_1 \underset{K_1}{=} A_6;\ A_1 \underset{K_2}{=} A_6;\ A_1 \underset{K_3}{<} A_6;\ A_1 \underset{K_4}{<} A_6;\ A_1 \underset{K_5}{<} A_6;\ A_1 \underset{K_6}{=} A_6;\ A_1 \underset{K_7}{=} A_6;\ A_1 \underset{K_8}{=} A_6.$$

According to obtained results of comparison A_1 and A_6, the relation $A_6 \geq A_1$ is fulfilled. Respectively A_1 is the dominant alternative and it is removed from further consideration.

2. Compare in pairs A_2 and A_3; A_2 and A_4; A_2 and A_5; A_2 and A_6 according to each criterion $f_1 \div f_8$.

$$A_2 \underset{K_1}{>} A_3;\ A_2 \underset{K_2}{<} A_3;\ A_2 \underset{K_3}{>} A_3;\ A_2 \underset{K_4}{<} A_3;\ A_2 \underset{K_5}{<} A_3;\ A_2 \underset{K_6}{>} A_3;\ A_2 \underset{K_7}{>} A_3;\ A_2 \underset{K_8}{<} A_3;$$

$$A_2 \underset{K_1}{=} A_4;\ A_2 \underset{K_2}{=} A_4;\ A_2 \underset{K_3}{>} A_4;\ A_2 \underset{K_4}{=} A_4;\ A_2 \underset{K_5}{=} A_4;\ A_2 \underset{K_6}{>} A_4;\ A_2 \underset{K_7}{=} A_4;\ A_2 \underset{K_8}{>} A_4;$$

$$A_2 \underset{K_1}{>} A_5;\ A_2 \underset{K_2}{<} A_5;\ A_2 \underset{K_3}{=} A_5;\ A_2 \underset{K_4}{<} A_5;\ A_2 \underset{K_5}{<} A_5;\ A_2 \underset{K_6}{>} A_5;\ A_2 \underset{K_7}{=} A_5;\ A_2 \underset{K_8}{>} A_5;$$

$$A_2 \underset{K_1}{>} A_6;\ A_2 \underset{K_2}{=} A_6;\ A_2 \underset{K_3}{<} A_6;\ A_2 \underset{K_4}{<} A_6;\ A_2 \underset{K_5}{<} A_6;\ A_2 \underset{K_6}{<} A_6;\ A_2 \underset{K_7}{>} A_6;\ A_2 \underset{K_8}{<} A_6.$$

According to obtained results of comparison A_2 and A_4, the relation $A_2 \geq A_4$ is fulfilled. Respectively A_4 is the dominant alternative and it is removed from further consideration. The alternative A_2 is added to Pareto set $P = \{A_2\}$.

3. Compare in pairs A_3 and A_5; A_3 and A_6 according to each criterion $f_1 \div f_8$.

$$A_3 \underset{K_1}{<} A_5;\ A_3 \underset{K_2}{=} A_5;\ A_3 \underset{K_3}{<} A_5;\ A_3 \underset{K_4}{=} A_5;\ A_3 \underset{K_5}{<} A_5;\ A_3 \underset{K_6}{=} A_5;\ A_3 \underset{K_7}{<} A_5;\ A_3 \underset{K_8}{>} A_5;$$

$$A_3 \underset{K_1}{<} A_6;\ A_3 \underset{K_2}{>} A_6;\ A_3 \underset{K_3}{<} A_6;\ A_3 \underset{K_4}{=} A_6;\ A_3 \underset{K_5}{=} A_6;\ A_3 \underset{K_6}{<} A_6;\ A_3 \underset{K_7}{=} A_6;\ A_3 \underset{K_8}{=} A_6.$$

Pairs A_3, A_5, and A_3, A_6 are incomparable in $\geq$relation, respectively, A_3 is added to the Pareto set $P = \{ A_2, A_3\}$.

4. Compare in pairs A_5 and A_6 according to each criterion $f_1 \div f_8$.

$$A_5 \underset{K_1}{<} A_6;\ A_5 \underset{K_2}{>} A_6;\ A_5 \underset{K_3}{<} A_6;\ A_5 \underset{K_4}{=} A_6;\ A_5 \underset{K_5}{>} A_6;\ A_5 \underset{K_6}{<} A_6;\ A_5 \underset{K_7}{>} A_6;\ A_5 \underset{K_8}{<} A_6.$$

The alternatives A_5 and A_6 are incomparable, respectively, they both are included to the Pareto set $P = \{ A_2, A_3, A_5, A_6\}$.

Thus, the resulting Pareto set has the next form $P = \{A_2, A_3, A_5, A_6\}$, respectively, the choice of OS should be made among these options.

Consider a situation where it is difficult for decision maker to determine the relative importance of criteria, which is quite common in practice. In this case, it is advisable to use the mathematical apparatus of the theory of evidence [169-172]. Evidence theory suggests instead of comparing individual alternatives, to select from the original set of analyzed options (frame of discernment) the subgroup, and then determine the degree of their preference within a given scale over the whole set of analyzed options (the expert determines for which subgroups or subsets of alternatives he/she can determine the degree of his/her preference, while the advantages of a particular subgroup of alternatives are equivalent to the problem of pairwise comparison of this subgroup with the whole set of alternatives). Next, the results of the expert survey are processed and the weights of the alternatives (basic probability assignments) are calculated, according to one of the rules of combination (for example, the Dempster's rule of combination) the combined basic probability assignments are determined. The values of belief and plausibility functions are calculated for all formed subgroups of alternatives which are determined, including subgroups consisting of one alternative.

Let us consider an example of the proposed approach based on the obtained Pareto set $P = \{A_2, A_3, A_5, A_6\}$. Two experts ($E_1$ and E_2) were involved to decision-making process, which are formed the following possible subgroups of options:

$E_1 = \{A_3\}, \{A_5, A_6\}, \{A_2, A_3, A_5, A_6\}$. $E_2 = \{A_5\}, \{A_2, A_3, A_5, A_6\}$.

For making pairwise comparison were used 0÷6 scale of preferences, where the value 0 indicates the impossibility to compare alternatives (incomparability); value 1 indicates the uniformity (equivalence) of the elements; a value of 6 indicates the absolute degree of preference. The obtained results in the form of truncated matrix of pairwise comparisons (9) are given in Table. 2.

Table 2.

Truncated matrix of pairwise comparisons of selected subgroups of alternatives

<table>
<tr><th>$\boldsymbol{E_1}$</th><th>$\{A_3\}$</th><th>$\{A_5, A_6\}$</th><th>$\{A_2, A_3, A_5, A_6\}$</th></tr>
<tr><td>$\{A_3\}$</td><td>1</td><td>0</td><td>4</td></tr>
<tr><td>$\{A_5, A_6\}$</td><td>0</td><td>1</td><td>6</td></tr>
<tr><td>$\{A_2, A_3, A_5, A_6\}$</td><td>1/4</td><td>1/6</td><td>1</td></tr>
<tr><td>$\boldsymbol{E_2}$</td><td>$\{A_5\}$</td><td colspan="2">$\{A_2, A_3, A_5, A_6\}$</td></tr>
<tr><td>$\{A_5\}$</td><td>1</td><td colspan="2">2</td></tr>
<tr><td>$\{A_2, A_3, A_5, A_6\}$</td><td>1/2</td><td colspan="2">1</td></tr>
</table>

Determine the weights of the selected groups of alternatives according to (10). The obtained normalized weights can be considered as the basic probability assignments of formed by experts groups of alternatives.

Expert E_1:

$$m_1(\{A_3\}) = \frac{4}{4+6+\sqrt{2}} = 0.35;$$

$$m_1(\{A_5, A_6\}) = \frac{6}{4+6+\sqrt{2}} = 0.53;$$

$$m_1(\{A_2, A_3, A_5, A_6\}) = \frac{\sqrt{2}}{4+6+\sqrt{2}} = 0.12.$$

Expert E_2:

$$m_2(\{A_5\}) = 0.67;$$

$$m_2(\{A_2, A_3, A_5, A_6\}) = 0.33.$$

Considering experts as independent sources of information, it is possible to calculate the combined probability assignments for all selected focal elements (subgroups of alternatives), using one of the rules of combination. Table 3 presents the results of the intersection of the formed focal elements.

Table 3.

Intersection of the formed focal elements

		***E*₁**		
		$\{A_3\}$	$\{A_5, A_6\}$	$\{A_2, A_3, A_5, A_6\}$
***E*₂**	$\{A_5\}$	$\varnothing$	$\{A_5\}$	$\{A_5\}$
	$\{A_2, A_3, A_5, A_6\}$	$\{A_3\}$	$\{A_5, A_6\}$	$\{A_2, A_3, A_5, A_6\}$

For the case $\{A_5\} \cap \{A_3\} = \varnothing$ the coefficient of conflict is:

$$K = m_1(\{A_3\})m_2(\{A_5\}) = 0.35 * 0.67 = 0.23.$$

Accordingly, $1 - K = 1 - 0.23 = 0.77$. Based on the date from Table 3, the resulting focal elements will take next form: $\{A_3\}$, $\{A_5\}$, $\{A_5, A_6\}$ and $\{A_2, A_3, A_5, A_6\}$.

Given the low level of conflict, let us determine the combined probability assignments based on the Dempster's rule:

$$m_{12}(\{A_3\}) = \frac{1}{1-K} m_1(\{A_3\}) m_2(\{A_2, A_3, A_5, A_6\}) = \frac{0.35 * 0.33}{0.77} = 0.15;$$

$$m_{12}(\{A_5\}) = [m_1(\{A_5, A_6\}) m_2(\{A_5\}) + m_1(\{A_2, A_3, A_5, A_6\}) m_2(\{A_5\})]/0.77 = 0.57;$$

$$m_{12}(\{A_5, A_6\}) = m_1(\{A_5, A_6\}) m_2(\{A_2, A_3, A_5, A_6\})/0.77 = 0.23;$$

$$m_{12}(\{A_2, A_3, A_5, A_6\}) = m_1(\{A_2, A_3, A_5, A_6\}) m_2(\{A_2, A_3, A_5, A_6\})/0.77 = 0.05.$$

Taking into account the obtained results, the next values of the belief (*Bel*) and plausibility (*Pl*) functions for each initial alternative A_2, A_3, A_5, A_6 were determined:

$$A_2 : \begin{cases} Bel(\{A_2\}) = 0; \\ Pl(\{A_2\}) = m_{12}(\{A_2, A_3, A_5, A_6\}) = 0.05. \end{cases}$$

Since alternatives A_2 and A_6 are obtained a combined probability assignments of 0, then $Bel(\{A_2\}) = Bel(\{A_6\})$ and $Pl(\{A_2\}) = Pl(\{Pl_6\})$, respectively.

$$A_3 : \begin{cases} Bel(\{A_3\}) = m_{12}(\{A_3\}) = 0.15; \\ Pl(\{A_3\}) = m_{12}(\{A_3\}) + m_{12}(\{A_2, A_3, A_5, A_6\}) = 0.20. \end{cases}$$

$$A_5 : \begin{cases} Bel(\{A_5\}) = m_{12}(\{A_5\}) = 0.57; \\ Pl(\{A_5\}) = m_{12}(\{A_5\}) + m_{12}(\{A_5, A_6\}) + m_{12}(\{A_2, A_3, A_5, A_6\}) = 0.85. \end{cases}$$

The above results show that the biggest value of the belief and plausibility functions has alternative A_5, therefore, it is optimal.

Conclusions. A methodology of synthesis of information technology of qualitative modeling of the problem of choosing the optimal OS of the enterprise, which is based on the mathematical apparatus of the theory of graph-dynamic, has been proposed. Methods of the theory of graph-dynamic systems are used for synthesis of a number of alternative variants of hierarchical OS of the enterprise by possible transformations of their structure for the further solving of various problems of their optimization.

The approach allowing to automate search and formation of optimum organizational hierarchical structures according to some criterion (for example, expenses for the maintenance of employees of the organization) using methods of graph-dynamic is considered. The examples considered in this paper illustrate the possible transformations of hierarchical OS in solving various problems of their optimization.

To solve the problem of multi-criteria optimization in order to choose the optimal OS of the enterprise, an approach is proposed, which is based on the integrated use of the procedure of Pareto set formation and methods of evidence theory. The proposed approach allowed to obtain a more formalized procedure of narrowing the Pareto set.

The mathematical apparatus of the theory of evidence allows to model the uncertainty in the judgments of experts or decision maker (removed the necessary requirement of "unambiguous" superiority of one alternative over another). A numerical example of finding the optimal OS of the enterprise is given. The obtained results allow to increase the quality and efficiency of finding optimal solutions.