9.2 Research of reliability of mechanisms of parallel structure of hexapod type

NOMENCLATURE AND GLOSSARY

OP, *IOP*, *INT*	indexis of the object state (operational, inoperable, additional intermediate)
n	the number of degrees of mobility of the MPSK
m	the number of variants of reliability distribution of the MPSK subsystems
i, j	$i=1,\ldots,n$; $j=1,\ldots,m$
K_R	readiness factor
$K_R(t)$	readiness function
K_{RS}	stationary factor of readiness
$P(t)$	product reliability function
$P_i(t)$	the value of the reliability the *i*-degree of mobility of the MPSC
P_P, P_N, P_D	the probabilities of the subsystem being in states *OP*, *IOP*, *INT* respectively
P_G , P_{SJ} , P_S	reliability indicators of subsystems hydraulic drive, spherical joint and spindle respectively
P^{dis} , P_{DO}	reliability indicators of the system specified in the design and its optimal value
$\lambda_j(t)$	the failure rate index for the distribution of the *j*-variant of the MPSK subsystems
λ_0	the value of the failure rate at $t \to \infty$
λ_p	failure rate during the transition from the state *OP* to the state *INT*
λ_D	failure rate during the transition from the state *INT* to the state *IOP*
λ_N	failure rate during the transition from the state *IOP* to the state *OP*
T_P	the average duration of work before the first failure
T_D	the average duration of damage accumulation
T_N	the average duration of the system recovery (inoperable state)
S	the integration parameter in the Laplace transform
S_1, S_2	the roots of the denominator in the Laplace transform
$\beta_1, \beta_2, \beta_3$	the constant coefficients in the Laplace transform
G_i	indicator of system reliability for the *j*-variant of the distribution of reliability norms between the MPSK subsystems
$W_j(R)$, $C_j(R)$	the economic effect and costs of using the *j*-option with the level of reliability *R*
R_1, R_2, R_3	rational distribution of subsystem reliability indicators
MPSK	mechanisms of parallel structure and kinematics
TS	technical system

Widespread production and use in modern industry of high-precision robotic machines and industrial platforms based on mechanisms of parallel structure and kinematics (MPSK) such as biglide, bipod, triglyde, pentapod, hexaglide, hexapod, etc. determines the rapid development of scientific research on technical level indicators and criteria for system optimization of relevant structures at the design stage [492, 493].

The reliability of such complex technical systems (TS) is generally limited by the reliability of the main subsystems (**drives**: electric, hydraulic, pneumatic; **transmissiones**: mechanical, electrical, hydraulical; **modules** of control and monitoring), as well as individual **"weak elements"** (bearings, seals, sensors of kinematic and dynamic parameters of the operation of the TS with aggregate-modular structures), which require statistical analysis, mathematical and simulation modeling at the design stage [494, 495].

The problem of structural reliability of the MPSK and its provision at the required level requires the solution of a number of scientific problems, namely:

- development of methods for determining the readiness function for typical MPSK schemes in the conditions of gradual accumulation of damage;
- structural analysis of the reliability of typical structures of the MPSK;
- substantiation of the recommendation on the optimal distribution of normative values of reliability indicators between the subsystems of the MPSK at the design stage.

The reliability of the MPSK depends on the reliability of individual subsystems, reliability indicators of which in design practice are accepted as basic normative parameters, the values of which are determined experimentally and provided during operation using appropriate lower level elements - "atomic elements", which are usually indivisible [496, 497]. Machine-robots and industrial platforms are designed for long-term use, and possible failures or delays in technological operations can be restored after repairs. Such TS are not used continuously, but include only for a certain time to perform certain complex high-precision and high-tech operations. The behavior of the restored TS is described by a random altered process with alternating periods of regulatory operation and recovery periods. A comprehensive indicator of the reliability of such TS is the readiness factor (*RF*), which reflects the probability $K(t)$ that the vehicle is operational at any time t, except for the planned periods during which the use of the vehicle is not provided [498, 499].

When assessing the reliability of the TS based on the MPSK, they are classified as objects, which are characterized by a gradual increase in the intensity of failures in the absence of reservations. In such cases, the formation of failure flows of subsystems can not be described as a Markov random process of transitions to various possible (operational or inoperable) states. To reduce non-Markov processes to Markov processes and further analytical description of vehicle reliability, it is proposed to introduce an additional intermediate state [497].

A directional graph of reliability states and transitions is constructed (Fig. 1). Hexapod-type MPSK (Fig. 2) is considered, in the structure of which three typical aggregate-modular subsystems (electromechanical drive of translational motion 1, rotating kinematic pairs 2, spindle mechanism 3) are distinguished, for which the failure time decreases increases [498].

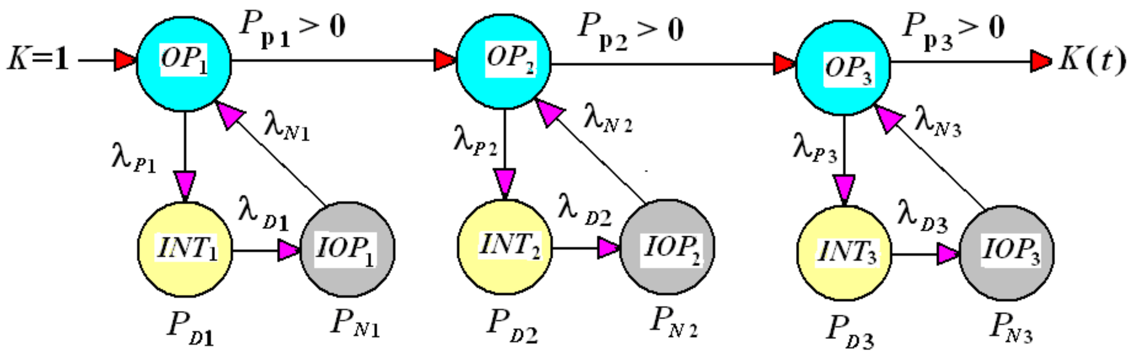

Figure 1. Graph of reliability states and transitions of connected subsystems 1, 2, 3 (indices correspond to the state: "*OP*" - operational; "*IOP*" - inoperable; "*INT*" - additional intermediate)

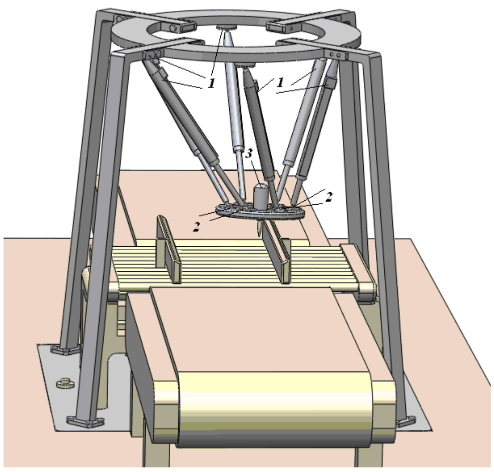

Figure 2. Model of the machine-robot on the basis of hexapod:
1 – electromechanical drive of translational motion;
2 – rotating kinematic pairs;
3 – spindle mechanism

MPSK subsystems (model elements) are different in failure and recovery intensities. If we neglect the scattering of the load between the subsystems, the load capacity of the elements are independent of each other, ie the corresponding failures can be considered statistically independent. The probability of failure-free operation in this case is determined by formula (1) in the same way as for non-recovery systems, ie

$$P(t) = P_1(t) \cdot P_2(t) \cdot \dots \cdot P_n(t) = \prod_{i=1}^{n} P_i(t). \tag{1}$$

Obviously, even with high reliability of each of the elements, the reliability of a sequential multi-element system is much lower. For example, for a hexapod with six degrees of freedom, given the equal reliability of the subsystem modules of each of the degrees of freedom at the level 0.95, the calculated indicator is equal. As a result of the analysis of the statistics of operational failures of the MPSK in the hexapod it was found that the "weak element" are the seals of the subsystem of the hydraulic drive of translational movements [500, 501].

Due to the introduction of additional intermediate states, it becomes possible to use in the models of Erlang reliability (from the **class of aging)** and hyperexponential (from the **class of youth)** distributions of operating time to failure. In particular, the Erlang distribution is a composition of a fixed number of exponentially distributed random variables. Functional analysis of the failure rate for the distribution of the *k*-order indicates that at the initial time the intensity $\lambda_j(t) = 0,$ and $\lambda_j(\infty) \to \lambda_0$.

By defragmenting the connected graph of the MPSK state system, the reliability graph of a separate subsystem is selected (Fig. 3). The transition from state "*OP*" to state "*INT*" is carried out with intensity $\lambda_{OP} = 1/T_P = const$ (T_P – average operating time before the first failure of the subsystem). The transition from state "*INT*" to state "*IOP*" is carried out with intensity $\lambda_D = 1/T_D = const$ const (T_D – average time of accumulation of damages), and transition from state "*IOP*" to state "*OP*" with intensity of failures $\lambda_N = 1/T_N$ (T_N – average recovery time of subsystem). After each recovery, the subsystem is considered completely renewed, ie its probabilistic characteristics become the same as in the new subsystem.

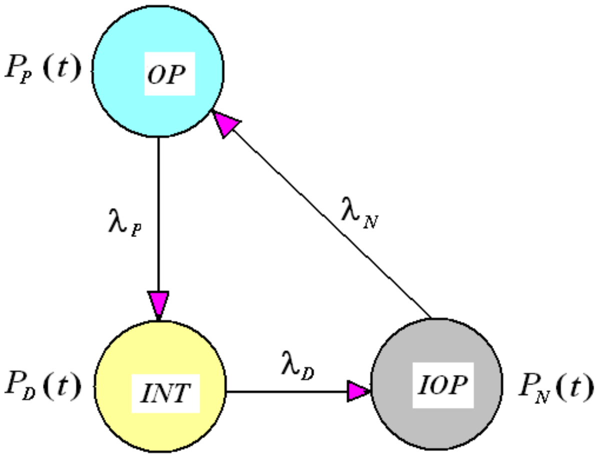

Figure 3. Graph of states and transitions of the subsystem of the MPSC drive: indices correspond to the state of the subsystem; $\lambda_P, \lambda_D, \lambda_N$ - corresponding intensities of transitions

On the basis of the graph (Fig. 3) the differential equations of dynamic balance for probabilities of constituent states in the form (2) are made:

$$\left.\begin{aligned} \frac{dP_p(t)}{dt} &= -\lambda_p P_p(t) + \lambda_N P_N(t); \\ \frac{dP_D(t)}{dt} &= -\lambda_D P_D(t) + \lambda_p P_p(t); \\ \frac{dP_N(t)}{dt} &= -\lambda_N P_N(t) + \lambda_D P_D(t) \end{aligned}\right\}, \qquad (2)$$

$P_p(t), P_D(t), P_N(t)$ – the probability of finding the subsystem in working, intermediate and inoperable states, respectively.

The normalized condition for the subsystem, which is in only one of the possible states (Fig. 3), is: $P_p(t) + P_D(t) + P_N(t) = 1$.

The given rationing condition corresponds to a closed graph of the reliability states of the subsystem. To analyze the dynamics of the behavior of the non-stationary readiness factor *RF* as the initial conditions for the general case, the condition is accepted that the subsystem starts working from a working condition, ie

$$P_p(0) = 1; \quad P_D(0) = 0; \quad P_N(0) = 0. \qquad (3)$$

To integrate equations (2) with the initial conditions (3), the method of Laplace transforms on the one-to-one correspondence of the original function $P(t)$ and its transformation $\varphi(s)$ in the form of (4) is used:

$$\begin{pmatrix} \lambda_p + s & 0 & -\lambda_N \\ -\lambda_p & \lambda_D + s & 0 \\ 0 & -\lambda_D & \lambda_N + s \end{pmatrix} \cdot \begin{pmatrix} \varphi_p(s) \\ \varphi_D(s) \\ \varphi_N(s) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \qquad (4)$$

In the phase space of transitions, the system (see Fig. 1) consists only of transitive (non-absorbing) states, so the first two of the three equations in system (2) are chosen for the solution and the rationing condition (3) is added to them. The equation in the form (5) is obtained:

$$\begin{pmatrix} \lambda_p + s & 0 & -\lambda_N \\ -\lambda_p & \lambda_D + s & 0 \\ s & s & s \end{pmatrix} \cdot \begin{pmatrix} \varphi_p(s) \\ \varphi_D(s) \\ \varphi_N(s) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}. \qquad (5)$$

Non-stationary *RF*, the probability that while in working order the subsystem will not go into failure is determined by equation $K(t) = P_p(t)$.

The Laplace transform based on equations (1) has the form (6):

$$\left. \begin{aligned} \varphi_p(s) &= \frac{\begin{vmatrix} 1 & 0 & -\lambda_N \\ 0 & \lambda_D + s & 0 \\ 1 & s & s \end{vmatrix}}{\begin{vmatrix} \lambda_p + s & 0 & -\lambda_N \\ -\lambda_p & \lambda_D + s & 0 \\ s & s & s \end{vmatrix}} = \frac{s^2 + s(\lambda_D + \lambda_N) + \lambda_N \lambda_D}{s\left[(\lambda_p + s)(\lambda_D + s) + \lambda_N(\lambda_p + \lambda_D + s)\right]} = \\ &= \frac{s^2 + s(\lambda_D + \lambda_N) + \lambda_N \lambda_D}{s\left[s^2 + s(\lambda_p + \lambda_D + \lambda_N) + \lambda_p \lambda_D + \lambda_p \lambda_N + \lambda_D \lambda_N\right]} \end{aligned} \right\}. \qquad (6)$$

In order to obtain the inverse Laplace transform, it is convenient to display function (6) as the sum of the type $a/(s+b)$, ie in the form (7)

$$\varphi_p(s) = \frac{\beta_1}{s} + \frac{\beta_2}{s - s_1} + \frac{\beta_3}{s - s_2} = \frac{s^2(\beta_1 + \beta_2 + \beta_3) - s\left[s_1(\beta_1 + \beta_3) + s_2(\beta_1 + \beta_2)\right]}{s(s - s_1)(s - s_2)}, \qquad (7)$$

where s_1, s_2 – the roots of the denominator in equation (6); $\beta_1, \beta_2, \beta_3$ – constant coefficients, which are determined by equating the numerators of equations (6) and (7):

$$\left.\begin{array}{l} s_1 = -(a_0 - b_0); \qquad s_2 = -(a_0 + b_0); \qquad a_0 = 0{,}5(\lambda_p + \lambda_D + \lambda_N); \\ b_0 = 0{,}5\sqrt{(\lambda_p + \lambda_D - \lambda_N)^2 - 4\lambda_p\lambda_D}; \quad \beta_1 = \dfrac{\mu\lambda_p}{\lambda_p\lambda_D + \lambda_p\lambda_N + \lambda_D\lambda_N} \end{array}\right\}. \tag{8}$$

$$\left.\beta_2 = \frac{\lambda_D + \lambda_N + s_1 + \dfrac{\lambda_N\lambda_D}{s_1}}{s_1 - s_2}; \qquad \beta_3 = \frac{\lambda_D + \lambda_N + s_2 + \dfrac{\lambda_N\lambda_D}{s_2}}{s_1 - s_2}\right\}. \tag{9}$$

After substitution (8) and (9) in equation (7) and transformations obtained:

$$\varphi(s) = \frac{\lambda_N\lambda_D}{\lambda_p\lambda_D + \lambda_p\lambda_N + \lambda_D\lambda_N} + \frac{\beta_2}{s + (a_0 - b_0)} + \frac{\beta_3}{s + (a_0 + b_0)}. \tag{10}$$

Going from the image (10) to the original, according to the Laplace transform, we obtain a function for **nonstationary** in the form

$$K_R(t) = \frac{\lambda_N\lambda_D}{\lambda_p\lambda_D + \lambda_p\lambda_N + \lambda_D\lambda_N} + \beta_2 e^{-(a_0 - b_0)t} + \beta_3 e^{-(a_0 + b_0)t}. \tag{11}$$

Accepting $t \to \infty$, non-stationary ***RF*** coincides to a certain value, the so-called **stationary *RF*** (Fig. 4), which is determined by the boundary transition

$$K_{RS}(t) = \lim_{t\to\infty} K_R(t) = \frac{\lambda_N\lambda_D}{\lambda_p\lambda_D + \lambda_p\lambda_N + \lambda_D\lambda_N} + \frac{1}{\lambda_p} \cdot \frac{1}{\left(\dfrac{1}{\lambda_p} + \dfrac{1}{\lambda_D} + \dfrac{1}{\lambda_N}\right)} = \frac{T_p}{T_p + T_D + T_N}.$$

The optimal level of reliability corresponds to the maximum of the objective function in the form

$$G_i(R) = W_i(R) - C_i(R),$$

where R – system reliability indicator, depending on the selected *i*-th option for the distribution of reliability standards between subsystems; ;

$i = \overline{1,n}$, n – number of options;

$W_i(R)$ – economic effect from the use of the *i*-th option with the value of the reliability indicator at the level of R;

$C_i(R)$ – the cost of ensuring the level of reliability at the level for the *i*-th option.

For each variant of distribution of norms of reliability the optimum decision is found on a condition:

$$\frac{\partial W_i(R)}{\partial R} = \frac{\partial C_i(R)}{\partial R}.$$

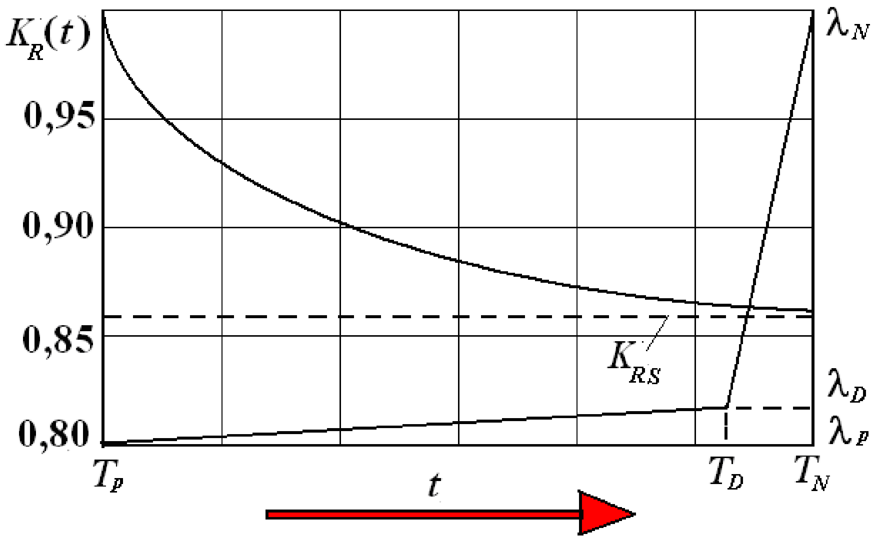

Figure 4. Graph of changes in the readiness function of the subsystem of the hydraulic drive MPSK with reliability parameters: $T_P = 4000\ h,\ T_D = 600\ h,\ T_N = 60\ h$

According to the assumption that the elements of the system fail independently of each other, ie the failure of each element leads to the failure of the whole system, we have inequality

$$P_1(t) \cdot P_2(t) \cdot \dots \cdot P_n(t) = \prod_{i=1}^{n} P_i(t) \geq P^{dis}, \tag{1}$$

where P^{dis} – the reliability of the system is specified during design.

To optimize the level of reliability of the subsystem of the hexapod hydraulic drive, the method of rational distribution of the reliability of the elements was used [497].

For the TS consisting of *n* elements, we rank the reliability of the elements in ascending order $P_1 \leq P_2 \leq \ldots \leq P_n$. Each of the indicators $P_1, P_2, \cdots, P_k$ in the design is increased to the same value P^{dis}, and since $P_{k+1}, \cdots, P_n$ do not change. The number k is chosen by the maximum value of j for which the conditions are met

$$P_j \le \left(\frac{P^{dis}}{\prod_{j=1}^{n+1} P_j} \right)^{\frac{1}{j}} = r_j\,; \quad P_0^{dis} \le \left(\frac{P^{dis}}{\prod_{j=1}^{n+1} P_j} \right)^{\frac{1}{k}}, \tag{12}$$

where $P_{n+1} = 1$ – accepted by definition.

The reliability of the system after finding P_0^{dis} satisfies conditions (12), since the new value of the indicator P^{dis} is equal to $\left(P_0^{dis}\right)^k \cdot P_{k+1} \cdot \dots \cdot P_n = P^{dis}$.

For MPSK type hexapod with six parallel mechanisms (rods) in the machine-robot there are three main subsystems (1 – hydraulic, 2 – hinged rod supports, 3 – spindle), and determined the need to comply with the condition:

$$P_G \cdot P_{SJ} \cdot P_S \ge P^{dis}.$$

Based on statistical values: P_G =0.97; P_{SJ} =0.99; P_S =0.95, obtained $P = P_G^6 \cdot P_{SJ}^{12} \cdot P_S = 0{,}97^6 \cdot 0{,}99^{12} \cdot 0{,}95 = 0{,}71$.

In order to determine the most effective ways to increase the design value of the reliability indicator of the hexapod to the level P^{dis} = 0.84, the rational distribution of the values of the reliability indicators of the subsystems is determined. First, it is assumed that k = 1, then by formula (12) is obtained

$$P_0^{dis} = \left(\frac{P^{dis}}{\prod_{j=1}^{n+1} P_j} \right)^{\frac{1}{k}} = \left(\frac{0{,}84}{0{,}89 \cdot 0{,}95 \cdot 1{,}0} \right)^{\frac{1}{1}} = 0{,}999.$$

As a result $P = 0{,}999 \cdot 0{,}89 \cdot 0{,}95 = 0{,}84$, this distribution of reliability of subsystems is not rational, because for subsystem 1, consisting of six independent modules of translational motion, it seems almost impossible and economically impractical to further increase the indicator from P_G = 0.97 to $P_G = \sqrt[6]{0{,}999} = 0{,}9999999$. From formula (12) determined rational distribution of reliability indicators for subsystems

$$r_1 = \left(\frac{0{,}84}{0{,}89 \cdot 0{,}95 \cdot 1{,}0}\right)^{\frac{1}{1}} = 0{,}999;\ r_2 = \left(\frac{0{,}84}{0{,}95 \cdot 1}\right)^{\frac{1}{2}} = 0{,}88;\ r_3 = \left(\frac{0{,}84}{1}\right)^{\frac{1}{3}} = 0{,}94.$$

As a result $P_1 < r_1$, $P_2 < r_2$, $P_3 > r_3$, the largest value of j is obtained $P < r$ in this case, provided that it corresponds to j = 2, so k = 2 is accepted and determined

$$P_0^{dis} = \left(\frac{0{,}84}{0{,}95 \cdot 1{,}0}\right)^{\frac{1}{2}} = 0{,}88.$$

The result means that the reliability indicators for subsystems 1 and 2 must be increased from 0.83 to 0.88 and from 0.89 to 0.94, respectively.

For subsystem 3, the reliability indicator can be left at the previous level. With the rational distribution of reliability indicators for the three subsystems of hexapod and the TS as a whole obtained

$$P_G = \sqrt[6]{0{,}88} = 0{,}98;\ P_{SJ} = \sqrt[12]{0{,}94} = 0{,}995;\ P_S = 0.95,$$

$$P = P_G^6 \cdot P_{SJ}^{12} \cdot P_S = 0{,}98^6 \cdot 0{,}995^{12} \cdot 0{,}95 = 0{,}84,$$

that is, the design requirements for the reliability of the MPSK have been met.

By introducing an additional intermediate state in the directional graph of states and transitions for typical MPSK schemes, a method for determining the readiness function in the conditions of gradual accumulation of damage has been developed. The algorithm of the structural analysis of reliability of the MPSK is developed, the example of substantiation of recommendations concerning optimum distribution of norms of reliability between subsystems of the MPSK at a design stage is resulted.