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2.3 Multichannel networks with periodic external load

Mass-media networks are an effective means of analytical modeling of information and computer networks, cellular networks, economic models (logistics, insurance, etc.), especially at the stage of their design, when it is necessary to obtain estimates of the main parameters of the network. Stochastic networks can be considered as a set of interconnected functioning nodes, each of which provides a certain service. The requirement for entering the network can be considered as a set of tasks that must be performed by components of the network in a certain order. When a requirement needs to be serviced at some node, it may be busy serving other requirements. In this case, it may take a certain place in the queue to this serving node, either interrupt the service request received earlier, or get a refusal. All these moments should be formalized.

Thanks to the theory of mass service, it is possible to model components (computers, data storage devices, routers, switches, modems), from which the real network, the parameters of which need to be evaluated, is composed. The user needs to know how to behave on the network under certain operating conditions. Detailed calculations are all the more necessary for special purpose systems, unique systems and networks.

In connection with the emergence of computer networks and the rapid introduction of data transmission networks, the scope of the theory of mass service has expanded considerably. First of all, its apparatus is used at all levels of organization of network structures in designing their topology, in developing protocols, in choosing switching methods and routing algorithms, in managing flows. The widespread application of the apparatus of the theory of mass service (TMS) is due to the simplicity and naturalness of modeling with the help of real networks and processes for processing them requirements.

Most authors, investigating the system of mass maintenance, put forward the condition that their parameters do not change over time ([148], [149], [151]). However, for many technical systems (elements of computer networks, computing systems, communication networks) this assumption is not always fulfilled. For example, as a result of the failure of one of the network elements, the intensity of the input stream may decrease as the requirements for this element are temporarily not generated. Parameters of requirements streams in real networks may experience changes over time for the following reasons:

- non-stationary incoming traffic to the network ([148], [150];
- changing route requirements that cause new or extinct existent threads at the network node's input (4]);
- failure or blocking of individual elements of the network, resulting in a decrease in the requirements flow on the input of other network components ([151]).

In addition, the functioning of the nodes of local computer networks, as well as the nodes of global computer networks of the type "Internet" (provider communication nodes, web servers, transmission stations, etc.) is described by mass service systems with the parameters that change at random moments of time ([152], [153]).

The inbound flow of requirements is usually Poisson due to the fact that it has the properties of regularity and the absence of aftereffects: the requirements are received for service one and the number of requirements received in one interval of time does not depend on the number of requirements for another period of time. The non-stationary flow is due to the fact that the flow rate may vary with time, which, for example, is typical for mobile networks throughout the day.

The main complexity of the analysis of mass-servicing systems in non-stationary mode, when the parameters of the SMO are time-dependent functions, consists of solving, as a rule, an infinite system of differential equations with variable coefficients.

Analysis of mass service systems with parameters that are functions of time develops mainly in two directions. One of them is a theoretical study of random processes in SMO and the development of numerical methods for calculating service characteristics. Appropriate methods for analyzing mass service systems with parameters that are time functions are also being developed. Often, heuristic assumptions are made.

Real network elements of communication systems are non-stationary objects. Their non-stationarity is conditioned, in particular, by changes in the load, the movement of network subscribers, failures and network components restoration, configuration changes, etc. The lack of adequate, if possible, simple, apparatus for research of non-stationary networks leads to the fact that networks are calculated on the maximum (peak) load.

Research of systems and networks of mass service with time-varying parameters is a complex mathematical task. So far, there are no universal methods (both approximate and numeric) that can be used to calculate the characteristics of systems and mass-servicing networks with variable parameters. So there is a need to develop such methods for at least certain classes of systems.

The proposed work is devoted to the study of multi-channel mass-servicing networks with a Poisson input stream (the only one for the whole network or autonomous for each node), the intensity of which depends on the time, and the different type of time distribution service requirements in the nodes. In particular, networks with periodic input flow are considered. The most common models are networks in which demand service time has an arbitrary distribution function.

The main problems considered are related to the approximation of some random processes describing the process of servicing the network, with the help of others - simpler and better investigated. This gives you the opportunity to count network quality functionality and solve relevant optimization tasks.

The obtained results allow to effectively solve problems of modeling, research and optimization of various classes of complex systems such as mobile communication networks, distributed computer networks, motor transport streams, call centers.

Markovian and non-Markovian models of stochastic networks. Mass service is a finite set of nodes - systems of service, among which circulating requirements, passing between nodes in accordance with the routing matrix.

When considering networks and mass service systems, their structure and algorithm of operation are specified, and the function of time interval distribution between successive delivery requirements, service time distribution function, number of serving devices, and the length of the maximum possible queue is specified. For networks, the number of service nodes is additionally indicated. In the theory of mass service, the Kendal symbolism is used to denote the SMO: *A/B/L/N*, where *A* and *B* determine the types of distribution functions for the input flow of requirements and service time, *L* – number of servicing devices, *N* – number of waiting places. Instead of A and B, you can insert characters from such a set: $\{M, E_r, HR, D, G\}$, where M means an exponential distribution (Markovian), E_r – Erlang distribution of order r (Erlangian), *HR* – hyper-exponential distribution of order *R*, *D* – degenerate distribution (Deterministic), *G* – distribution of the general type (General).

Under the network of service in the network understand the system of mass service, consisting of L ($0 < L \le \infty$)the same service and buffer (queue) volume N ($0 \le N \le \infty$). At $L = 1$ the node is called single-line (single-channel), with $L = \infty$ – multichannel.

For those requirements that have completed the service in i - so the nodes are asked the probabilities of transitions to other nodes and the probability of leaving the network. Probabilities of transitions between nodes form a matrix of routing.

The external flow of requirements entering the network is determined by the coherent distribution of random variables, which are sequential intervals between the moments of receipt of claims. If these random variables are independent in aggregate, then such a stream is called a stream with a limited aftereffect, and for its definition it is sufficient to specify a set of distribution functions, where τ_k – the time interval between receipt $(k-1)$ and k - requirements. An important role in the theory of systems and mass maintenance networks is played by the recurrence stream, for which the Poisson flow is a special case of a recurrent stream $x \ge 0$, where is the flow rate.

In the theory of stochastic systems and networks, it is often assumed that the service time at the nodes of the network is exponentially distributed. This is due to the fact that requirements are processed at a constant rate, and the length of the requirement is a random variable distributed according to the index law. At the same time, most models are based on time-consuming process requirements and distribution of service time. However, in practice, there are real cases where the intensity of the input stream depends on time. This aspect encourages the use of heterogeneous time-based processes for modeling real networks. Natural is the assumption of the periodic dependence of the intensity of the input flow from time to period of one day, week or month. Such models are much less explored than networks and systems with sustained-intensity flows. Some of the results can be found in the works [149], [155], [157].

D. Hayman and W. Witt [151] reviewed the system $M_t \mid M \mid c$ and showed that, given the periodicity of the input stream, the process that describes the length of the queue is asymptotically periodic. The authors also obtained results regarding the strong asymptotic stability of the process describing the system. They set the ratio between the average number of requirements and the average time required by the system.

L. Brouwer ([150], [154]) studied the type of SMO $BMAP \mid M_t \mid c$ (batch Markovian arrival processes), the input of which receives a periodic BMAP flow, the service time is distributed exponentially, and the intensity of service varies in time from time to time. Conditions of stability are proved in terms of intensities of receipt and maintenance. Special class is selected $BMAP \mid M_t \mid c$ - systems, in which it is possible to simplify calculations to the level of homogeneous SMOs.

The SMO with one maintenance device and the periodic intensity of service is studied in the monograph by M. van Eenig [152]. For such systems, the time axis is divided into intervals of equal length, called cycles. During the cycle the device serves different queues of requirements. The order in which he "visits" the queues is the same for each cycle.

G. Falin [154] developed an analytical approach to the diffusion approximation of the queue in the system $M_t \mid G \mid 1 \mid \infty$ with periodic input flow. The author uses the principle of averaging: the periodic queue is approximated by the corresponding stationary queue with the intensity of the input $\lambda = \lim_{t \to \infty} \frac{1}{t} \int_0^t \lambda(u) du = \Lambda$. We find conditions under which this approximation gives a good result. It is shown that the principle of averaging takes place for SMO in the conditions of high loading in the sense of convergence of the non-stationary process to the stationary one, which is a Brownian motion with reflection.

It should be noted that stochastic networks with periodic input flow have practically not been investigated. A lot of work is devoted to the analysis of mass service models in conditions of overload mode of operation. An overloaded mode means that the characteristics of the model depend on some parameter n, and with $n \to \infty$ The average system load is one unit (or even more than one). The study of overloaded mode has a long history and distinguishes several areas oriented to different classes of models of mass service. Many authors have been dealing with incoming recovery processes, independent maintenance hours, and maintenance processes that are independent of the current queue and time length. For this case, the convergence of the characteristics of the normalized queue length to the solution of the differential equation or to the Brownian motion with reflection in the corresponding boundaries is proved. The methods of analysis in these works mainly use the functional central boundary theorems for the input stream, the process of maintenance and the routing process.

Another direction contains an analysis of Markov models of mass service, in which there is a dependence on the state of the system. In this case, the analysis method is mainly based on martingale technique. Using this technique, [152] proved the convergence of the maintenance process in the network $\left[M \,|\, M_Q \,|1|\, c_n\right]^r$, which depends on the length of the queue, under the conditions of the overload mode of operation before the diffusion process. For networks of type $\left[M \,|\, M_Q \,|1|\, \infty\right]^r$ in the works [152] the convergence to the diffusion process is investigated. Markov models, which depend on time, are considered in particular in the works [157], [158]. Some results for incoming processes depending on the network status and total distribution of service times are given in [159].

Not only for the development of the theory, but also from the practical point of view, it is important to examine the processes of service in general assumptions both in relation to the flow of requirements and in relation to the distribution of service time.

It is commonly believed that the input flow with a very good approximation can be considered Poisson, at least in the tasks of telephony and tasks of information processing. This idea is based on the limit theorem on the total flow generated by many plurality of low intensity. Therefore, it is first and foremost important to study models in which the duration of service has an arbitrary distribution.

Many works devoted to this area of research of stochastic systems and networks. They got a lot of interesting results regarding the values of the main characteristics of the operation of the model. Mass-servicing networks with arbitrary service time, in particular under conditions of overload mode of operation. Interesting results from the research of Markov and non-Markov systems and mass-servicing networks with unlimited number of service devices have recently been received by a group of Tomsk scientists. They proposed and developed an approach called the sifted flow method ([151], [152], [153]). This method allows the problem of research of the non-Markov system (network) service with an unlimited number of devices for servicing to reduce the problem of the analysis of a non-stationary flow that is marketed. In particular, [154] presents the study of an open-source mass service network with input high-intensity flow requirements called the High Intensivative Markov-Modulated Poisson Process (HIMMPP) with unlimited number of servicing devices in nodes, Markov routing, and recurrent maintenance of requirements in conditions of increasing intensity input stream An asymptotic analysis of this network is carried out. Using the method of dynamically sifted flow for such networks, the authors obtained an expression for calculating the average values of the number of devices employed in the node s of the described network. It is also established that under the conditions of unlimited increase of the intensity of the input stream, the multidimensional distribution of probabilities of the number of employed devices in the nodes is approximated by a multidimensional normal distribution. In this paper we obtain the characteristics of this distribution in the form of a vector of mathematical expectations and the covariance matrix.

Basic Methods and Directions of Stochastic Network Research. The basic approaches to the study of mass service networks are based either on the direct method of finding expressions for the probabilities of the network conditions using the technique of constructing equations of the local balance, or on the method of constructing recurrent equations for the mean values. The above methods allow us to find the exact solution for multiplicative or, as they are often called, locally balanced networks, the stationary probabilities of states of which have a multiplicative form. For these networks, such an interesting fact was established: the stationary probabilities of the network states have a multiplicative form: in the stationary mode, the distribution of the number of requirements at the nodes of the network is independent, and their product is the stationary distribution of the entire network. This fact became the basis for further analyzes of the future of the MEMO of the general type, as well as the development of effective algorithms for the calculation of the characteristics of stochastic networks.

The search for the conditions for the existence of a multiplicative form of stationary distribution and in the last decades is reduced mainly to operation with the local equilibrium equation (see, eg [151], [155]) This leads to the expansion of known results on a more general-looking network. The disciplines of service are generalized, depending on the intensity of the input stream from the state of the network and / or time, etc.

In recent years, a number of results have been obtained, which greatly expands the possibilities of analysis of mass service networks. The main methods of analysis, which today are rather complete character, are distinguished.

Here are some basic research methods.

The Martingual approach based on the semimartingal representation of the investigated process and the use of a well-developed apparatus for the theory of martingales ([161],[163]) for the diffusion approximation of service networks, for solving filtration and optimal control problems, and also for statistical analysis of networks.

The recurrence approach ([150]), which consists in the recurrence relations for the characteristics of the system under consideration, and for the further asymptotic analysis, the boundary theorems on the convergence of stochastic recurrence schemes to the solution of stochastic differential equations (SDE). This method is used to study a wide class of service systems with repeated calls and in the study of local area networks with multiple access protocols. G. Falin [154] introduced a class of random switching processes, which is a convenient means for describing the asymptotic behavior of a broad class of stochastic models. The switching process, the author calls the service process, which can be represented as a semi-Markov process. As a rule, the random environment in which the service model operates is considered as an external process, which is not influenced by the behavior of internal processes. The theorems of the type of the principle of averaging and diffusion approximation are proved in [149], [150], [151], and some classes of systems and mass service networks that can be represented by switching processes are considered. Asymptotic behavior has been investigated for these models, in particular under conditions of low-intensity input flows and under over-mode conditions.

The method of diffusion approximation in work [152] is used for analysis in the conditions of high load of open models of computer networks with arbitrary interval distributions between the moments of receipt of requirements to the network and the duration of their service. In this paper, common closed exponential stochastic networks are also investigated, when the number of requirements in them indefinitely increases. For such networks, conditions for congestion are formulated, in which the proposed deterministic and Gaussian approximations for the process of queue lengths are proposed. The authors managed to obtain an asymptotic schedule for the main indicators of the functioning of the networks.

Non-stationary mass service systems (SMO) also studied in works A.I. Seyman, who describes the process of serving claims in terms of birth and death processes and examining the SMO with the help of an advanced device for these processes. The proposed method is based on the logarithmic norm of the linear operator and the special transformation of the cut-off matrix of intensities of the Markov chain under consideration. This method is applicable to the class of Markovian SMO. In particular, [163] is devoted to the estimation of the convergence velocity for different types of inhomogeneous Poisson flows with a specific form of heterogeneity. Namely, when the intensity of birth and death (the intensity of the input flow and service) is asymptotically periodic functions of time.

The given different approaches allow the experts to choose from them the most convenient and effective in a concrete situation. The data of numerical and simulation modeling of networks demonstrate the fact that the results of approximative formulas are quite close to precise with a sufficiently large load.