SECTION 10. THEORY AND METHODOLOGY OF PROFESSIONAL EDUCATION

DOI: 10.46299/ISG.2024.MONO.PED.1.10.1

10.1 European education: myth or reality. Comparative analysis of mathematics teaching in the secondary and higher education system in Germany and Ukraine

The searching for better living conditions, better paid jobs, new opportunities and, as a consequence, sharp increasing of the migration flows has become one of the main trends in the development of modern humanity.

According to the latest statistics, the undisputed leader of migration is the United States (about 49.8 million people live in the territory), then follow Saudi Arabia and Germany (about 12.2 million people) and Russia (about 11.2 million people) [308]. People change their country, their way of life, their habits for different reasons, and the purposes of migration are quite different, but conditionally migration flows can be divided into three main groups.

The first group are those people who consider the country are transit migrants. For example, programmers who see the USA or Australia as their final destination, but have to live and work in Poland, Germany, France, etc. for some time.

The second group are people (refugees) who are forced to leave their homes at the moment due to severe external circumstances, and for them the place, time and country of immigration are often irrelevant.

The third group is those people who do it consciously, moving their families and, as they say, choosing the country forever. Therefore, this category chooses the place of their future residence quite carefully, realizing that not only they will have to live and work here, but also their children will probably become citizens of the new country.

And if we are talking about European countries, Germany is very often the choice of the people. Thus, about 50% of people who immigrated to Germany from Russia, Ukraine and Belarus in the last 2-3 years stayed in Germany, and the rest of them leaved the country for some personal reasons. [309].

There can be many explanations why Germany is the country of choice. These include a developed economy, a high level of social protection, geographical location

(the opportunity to visit relatives and friends often), a huge labor market, etc.
And one of the most important reasons for moving to Germany is the opportunity to give the children a good European education.

Let us compare European education with education in the post-Soviet countries to answer the question "Is European education really much better?". The comparison will be made on the example of mathematics teaching in secondary schools at all levels and Universities in Germany (Berlin-Brandenburg) and Ukraine. Mathematics as a discipline for this study was not chosen by chance.

Firstly, it is one of the basic disciplines and its "language" is universal all over the world, and it is a basic discipline in both school (Germany and Ukraine) and in higher education too.

Secondly, mathematics is a science of numbers more than words, and therefore it is probably the easiest subject to adapt to it in new conditions. Our own teaching experience in both countries: Ukraine and Germany allows us to conduct such a study. Let us conditionally divide the period of schoolchildren's education in Germany into three parts: 7-8 grades; 9-10 grades, 11-12 (13) grades [310].

Let's look shortly on education level in elementary school (Grundschule), this is education up to 6^{th} grade. Here all children study according to the same programs, and the purpose of such education is to obtain basic knowledge. More successful pupils go on to the Gymnasium, those who do not show more as average knowledge, continue their studies in the secondary school (Sekundarschule).

If we talk about 5^{th} -6^{th} grades of mathematics education in Germany, an average child with a good knowledge of German language can without any relative difficulties enter into the system of German level of mathematic. The methodology of presentation of material is similar at this stage of learning in German schools to the methodology of teaching material in Ukrainian schools. There are some differences, but they are insignificant.

It should be noted that the level of training in mathematics in German schools is much lower at this stage, and any child from the post-Soviet space will be bored in classes very simply. One of the significant pluses at this time will be the opportunity

to learn the language of the country without significant distraction from the subject material. In general, among the disadvantages of "German math" of this level can be distinguished:

- the material is easy, the topics are periodically repeated, that produces the filling of endless this process, leads to some tiredness and losing of interest in the subject by cleverer children;

-not quite successful sequence of material presentation. For example, in 6th grade, at the beginning of the school year, pupils study addition and subtraction operations with the fractions, the process of conversion of improper fractions into mixed fractions. After finishing the study of this topic, the textbook offers a topic on geometry as next, and then it goes back to the previous topic “The Fractions”, and it’s proposed to study the division and multiplication as operations with the fractions. At the same time, pupils have already forgotten what they have learned in the topic "Fractions" before, and they lose the integrity of perception of the topic.

But positive facts should also be noted. Thus, there isn’t any total control over the sequence of topics in the system of secondary education, and therefore the teacher can change places and combine the necessary topics how he/she likes. In addition, German math textbooks are filled with a lot of tasks with practical content and application of mathematical calculations in everyday life.

As mentioned above, there is a division of children into Sekundarschule and Gymnasium pupils after 6th grade of study.

We will not stay a long time on the level of mathematic in Sekundarschule. Let us only note the fact that the level of teaching or even the level of knowledge and requirements is by definition very low, because this level is not oriented towards further studies in a higher education institution. And if it is so important for the child to continue his education at university, for example, then it to be in the Gymnasium on the time is very important for him.

University education is also possible after Sekundarschule, but it requires a lot of effort and time. For a better understanding of the situation, it shown some examples that demonstrate the level of material presentation in the Sekundarschule.

For example, there are such topics as "Rational numbers", "Expressions and equations". (Rationale Zahlen, Terme und Gleichungen), in the 7th grade of Sekundarschule.

On the topic “Rational Numbers” (Rationale Zahlen), the pupils have only deal with the operations and examples look like these: $-(-22)+31=$.

They solve equations of the form: $6x+5=2x+21$ from the topic "Expressions and Equations" (Terme und Gleichungen) [311].

In the 8th grade of Sekundarschule completely all topics are repeated, and the examples are of the same level as in the 7th grade, or maybe a little more complicated. Here is an example of an equation from the same topic "Expressions and Equations" (Terme und Gleichungen), but from 8th grade textbook. Pupils are asked to solve the equation: $4y+6=3y+14$ [312].

Thus we can see that the distance between Sekundarschule and Gymnasium only increases with time, and the principle of "let's stay at Sekundarschule for a while, improve the language and move on" doesn’t work in the 99% of the time, and it’s a result of my own experience. There is a mechanism of transferring a pupil from a Sekundarschule to a Gymnasium, but it isn’t always possible after any year of study and this process is accompanied by passing transfer exams.

This division is a huge disadvantage in the German education system. On the one hand, it can be said that in this case weaker children don’t prevent better and cleverer children from learning, and this is a positive thing.

But there are situations when due to some external reasons, for example, children of emigrants, when they have a good level of the subjects but don’t know the language sufficiently, and as result they end up in a second school (Sekundarschule), or if a local pupil stops learning well due to some situations, it would be very difficult for him/her to change the format of education after the sometime.

In this case, if we compare both systems, the format of the old post-Soviet system is a huge advantage. Because the curriculum was unified, but there were strong and weak classes, and at any time a pupil had the opportunity to be involved in the learning process. Let us stay more on the level of mathematics teaching at the

Gymnasium. As mentioned above, the main advantages of German mathematics education are the large number of text problems of applied value. Moreover, many of them contain real data and the results of calculations can be applied in practice.

A positive factor is also the availability of a large number of hours for studying the subject up to 6 hours per week. However, there are also a significant number of disadvantages that should be taken into account and known for further studies and successful career. Let us highlight the main ones in general:

- the absence of division of mathematics into algebra geometry. Geometry is presented in fragments, and doesn't cover the whole volume of material, which is necessary for its deep enough understanding;

- the structure of the material is very badly, the presentation of subject is very specific. Thus, very often, as it was mentioned above, there is no consistent connection between the topics. There is too much presence of different level examples.

For example, very easy equations or tasks may be given, and the next ones will be very difficult, and vice versa. Very often, textual problems come first, which still require a clear understanding of the topic itself, and preliminary solving of examples, because solving a problem is first building a mathematical model, and only then solving it. Not always is working when the problem is shown, then the methods are attempted for its solving and only then the methods are chosen. As a consequence, such kind of thinking or solving is for many students is very difficult because they are neither researchers nor students;

- there isn't no theoretical material in the textbooks often. And the methodology of presentation of what is available leaves to wish much to be better. There aren't such important words as "theorem", "axiom" at all. Many mathematical statements are given as statement and aren't proved;

- a very strong, sometimes excessive, orientation towards independent work;

- significant differences in the level of preparation between the different parts (regions) of Germany (Bundesländer). For example, the levels of knowledge in Berlin-Brandenburg and Bavaria are different dramatically, so the chances of apply to study to the same university for pupils from different regions (Bundesländer) will be

completely different with the same subject score.

Let's take a closer look at the programs of study. Let's start with 7^{th} -8^{th} grades. As it has already been mentioned, German mathematics has two main differences inside of it: the methods of the materials presentation and its content.

Thus, at the beginning of the year (7^{th} grade) pupils study the topic "Direct and inverse proportionality" (Proportionale und antiprortionale Zuordnungen), the topic occupies about 48 pages of the textbook out of 250. It contains a huge number of one-type tasks, most of which only duplicate each other and not any more [313].

Although these topics could be an introduction to the topic "Linear Function", and take no more than two or three hours. At the same time, when a since the definition of "Inverse Proportionality" is given, it is reasonable to at least give the definition of "Hyperbola". But this doesn't happen.

The topic "Linear Function and its Graph" will be studied in exactly in one year, and most pupils simply don't understand the connection between these definitions, and forget what they have learned. Further, if we stick strictly to the program, the next topic from the textbook is "Ration Numbers" (Rationale Zahlen). This topic is represented by a bunch of simple tasks, which practically correspond to the level of the Sekundarschule.

Here is one of the tasks where you have to compare numbers, what is greater and what is less: "Übertrage in dein Heft und ersetze richtig durch > oder < $1)-6 \quad -8;\quad 2)0 \quad -4$." You can see how easy these tasks are.

The same section pays little attention to the modulus of a number. Moreover, there are very few tasks on topic "Module", and they are rather primitive.

Here is an example. it is necessary to compare values: "Übertrage in dein Heft und ersetze richtig durch > oder < $1)7 \quad |-3|;\quad 2)|95| \quad 45$ **"**.

Even the solution of equations of the form: $|x+3|=2$ causes difficulties for pupils, and besides, the types of equations, or examples of such level are simply absent [313].

Such kind of topics as "Areas of Plane Figures" ("die Flache des Parallelogramms, des Dreiecks, des Trapezes, des Vierecks") are included in the

educational process at the end of the 7th year of study. The same are studied in the 8th grade of Ukrainian school, but the level of presentation of material is based on the simplest tasks in German school, and it cannot be said that that part of geometry will correspond to the level of geometry of the 8th grade of schools of the post-Soviet space.

Thus, with a sufficient level of mathematical preparation, it isn't difficult to "catch up" with the missing material (it is presented in these classes quite simply). The only problem that may actually arise in 99% of cases, which simply needs to be paid attention to in advance, is the problem of translation and perception of German mathematical terminology, and taking into account a large number of text problems, which requires a certain level of language training. And the further one goes, the more complicated this process becomes.

Very often there is a complete discrepancy between German mathematical definitions and English ones (despite the fact that the languages belong to the same language group), and secondly, the impossibility of "direct" translation of some concepts.

For example, one of the 6th grade problems had to be solved using Dreisatz, and only the shown solution to the problem made it possible to understand that this is how the usual proportion of the form: $\frac{a}{b}=\frac{c}{d}$.

Such international words as "modulus" and "radians" are translated in German as "der Betrag" und "das Bogenmass". The area of a figure is denoted by the letter A, and the perimeter by the letter U, instead of the usual S and P. An interesting formulation is the word "Augensumme", which directly translates as "sum of eyes", but it denotes the sum of digits that fall on the dice during some tasks from the topic "Probability Theory" And there are many such examples.

It is also necessary to pay attention to the very specifics of material presentation. Thus, for example, if it is necessary to construct the equation of a line through two points in one of task, German teachers use the following methodology.

Thus, the equation of a linear function is presented in the form: $y=mx+b$. The given points are plotted on the plane $A(x_1;y_1)$; $B(x_2;y_2)$: the value of slope is

found: $m = \frac{y_2 - y_1}{x_2 - x_1}$. The sign of slope (whether the function is decreasing or increasing) must also be taken into account.

After the value of slope is found, one from two points is substituted into the original equation $y = mx + b$ an the value of the coefficient b is found.

Of other way, some definitions can be absent in German mathematics. For example, there is the definition of Zeros of a Function (where $y = 0$): “Nullstelle”, but there is no any really name of zeros of an argument(where $x = 0$).

The definition of point where $x = 0$ is defined in German math textbooks as follows: „Der Y- Achsenabschnitt ist der Y- Wert, an dem der Graph der Funktion die Y- Ache scheidet“.

Although the notion of “Nullstelle” can be found in another sense. Thus, for the function $f(x) = \frac{(x+3)(x+1)}{(x-2)(x-3)}$, the points $x = 2, x = 3$: are the zeros of the denominator, then they are “Nullstellen des Nenners” and the points: $x = -3, x = -1$: are the zeros of the numerator or “Nullstellen des Zählers” in German .

But if, for example, we change the condition of the task not so much and it looks like this: "Finde den Definitionsbereich der Funktion" (find the domain): $f(x) = \frac{(x+3)(x+1)}{(x-2)(x-3)}$, then in this case the zeros of the denominator (Nullstellen des Nenners) are the points of the function's discontinuity (Polstelle) or the points where the function is undefined.

The 8th grade program practically duplicates the 7th grade program, many topics are given at the same level, some in a slightly more complex form. Only a topic as "Fundamentals of Probability Theory" is added. I would like to especially note: at this level of education the material contains really a lot of practical tasks, but its chaotic presentation, leads sometimes to a complete misunderstanding of the logical connection between the topics. And the level of complexity of tasks leaves much to be desired.

So, for comparison, let us give two tasks on the same topic "Systems of linear equations", (Das lineare Gleichungssystem), these two topics are studied in the 8th grade of both German and Ukrainian schools, the tasks were selected as belonging to the category of complex, which are given in mathematics textbooks of both countries. Thus, the "complex" system proposed for pupils of the German school looks like this:

$$\begin{cases} y+7=5x-2 \\ 4x+3y=-8 \end{cases}$$ [314].

The task, corresponding to it on the topic "complex" system of linear equations from the algebra textbook of the 8th grade of Ukrainian school has the following form:

$$\begin{cases} \frac{3x}{5}-2y=5 \\ x-\frac{3y}{2}=6,3 \end{cases}$$ [315].

Here is another example from the topic "Simplifying Expressions". One of the examples, which can be classified as complex, from the textbook of German math 8th grade: $-a(3b-a)-(a+b)(a-b)$.

And here is the level of a more complex example from an 8th grade algebra textbook, Ukraine: $\frac{3a+15b}{a^2-81b^2}:\frac{4a+20b}{a^2-18ab+81b^2}$.

Thus, at this stage, with a little preliminary preparation of "mathematics in German", taking into account the peculiarities of the discipline in German schools, the future pupil, provided that he/she has done well in the subject in his/her country and understands the "mathematical" side of the issue, can quickly and painlessly enter the system of German secondary education. But it is necessary to understand that the level of mathematical training can be significantly lower, and this depends not only on the school itself, but also on where it is located in the full sense of this word.

Let's consider the structure of studying mathematics at the level of 9th -10th grades. As an example, take the topic: "Parabola" [316].

It begins with the usual study of the square function and its graph. Then, we look at functions of the form: $y = x^2$, and their graphs, which is quite logical, and we pass to the equation of a parabola in the form: $y = a(x - x_s)^2 + y_s$, which is literally called "the equation of a parabola in the form with a vertex" (Die Gleichung der Parabel in der Scheitelpunktform).

It should be noted that the methodological presentation of this topic is quite logical, after that pupils study two types of parabola equation: $y = ax^2 + bx + c$ - general parabola equation (gemeinsame Form der Parabel) and parabola equation in form $y = a(x - x_1)(x - x_2)$ (faktorisierte Form der Parabel). After this only follows the transition to the topic "Quadratic Equations." Let us stay on such kind shortcomings in the presentation of the topic which we would like to emphasize and which significantly both confuse and complicate the material.

Suppose there is a general equation of a parabola, for example: $y = 2x^2 - 5x + 3$, we need to find the coordinates of its vertex. In the algebra course of Ukrainian textbooks, pupils look for the coordinates of the vertex (x-axis value) with using the formula: $x = -\frac{b}{2a}$ and immediately get the answer: $x = \frac{5}{4}$,to find the value on the y-axis coordinate, the value of the X coordinate is simply substituted into the equation of the parabola: $y = 2x^2 - 5x + 3 = 2\left(\frac{5}{4}\right)^2 - 5 \cdot \left(\frac{5}{4}\right) + 3 = -\frac{1}{8}$ .So the Ukrainian pupils gets the coordinates of the vertex of the parabola: $\left(\frac{5}{4}; -\frac{1}{8}\right)$.

A German pupils doesn't know the formula for the coordinate of a vertex on X: $x = -\frac{b}{2a}$. For them it's necessary from general equation in the form $y = ax^2 + bx + c$ to arrive to the form: $y = a(x - x_s)^2 + y_s$ by complete squaring, where $(x_s; y_s)$ - are the coordinates of the vertex.

Let us give, as an example, the course of calculations in this case:

$$y = 2x^2 - 5x + 3 = 2(x^2 - \frac{5}{2}x) + 3 = 2(x^2 - 2 \cdot \frac{5}{4}x + \frac{25}{16} - \frac{25}{16}) + 3 = 2(x - \frac{5}{4})^2 - \frac{1}{8}$$

Thus, the pupils arrive the form: $y = a(x - x_s)^2 + y_s$, where $(x_s; y_s) = \left(\frac{5}{4}; -\frac{1}{8}\right)$.

Of course, the answers are the same, but the purpose of math is to teach logic, to form the ability for choosing the most comfortable and fastest way of decision, to act more rationally, not only to get a right answer.

Here is another example. For the same parabola given by the equation: $y = 2x^2 - 5x + 3$ and we need to find the zeros of the function (Nullstelle). What would a Ukrainian pupils do? In the algebra course is studied the topic of quadratic equations before, and they would solve the quadratic equation: $2x^2 - 5x + 3 = 0$. Obtained the roots: $x_1 = 1; \quad x_2 = \frac{3}{2}$. Then the zeros of the function (Nullstelle) would be written as $(1;0); \quad \left(\frac{3}{2};0\right)$. But in the course of German mathematics the topic "Quadratic equations" is studied later, and therefore here the solution of this problem would look like this: $y = 2x^2 - 5x + 3 = 2(x - \frac{5}{4})^2 - \frac{1}{8}$ -first we allocate the complete square. Then, when $y = 0$ we find the solution of equation: $2(x - \frac{5}{4})^2 - \frac{1}{8} = 0$.

$$2(x - \frac{5}{4})^2 - \frac{1}{8} = 0 \Rightarrow (x - \frac{5}{4})^2 = \frac{1}{16};$$

$$x - \frac{5}{4} = \frac{1}{4}; \quad x - \frac{5}{4} = -\frac{1}{4}.$$

-get the answers: $x_1 = \frac{6}{4} = \frac{3}{2}; \quad x_2 = 1$.

And write the zeros of the function as: $x_1 = 1; \quad x_2 = \frac{3}{2}$. Of course, for a good mathematical preparation it is necessary to know all the methods and ways of solving the given task. But, the emphasis should be shifted to the choice of finding the most optimal and rational solution.

In the same way, the task of going from the equation of a parabola in the form

$y = ax^2 + bx + c$ to the form $y = a(x - x_1)(x - x_2)$. Pupils are asked to first extract the full square, reduce to the form: $y = a(x - x_s)^2 + y_s$.

Then solve the quadratic equation: $a(x - x_s)^2 + y_s = 0$, find its roots, which will be the zeros of the function, and only then write the equation in the form: $y = a(x - x_1)(x - x_2)$.

It is not taught that there is a concept of "root of a polynomial", or at least, the fact that if a quadratic equation has a solution, it can be written in the form: $ax^2 + bx + c = a(x - x_1)(x - x_2)$, where $x_1; x_2$ are the roots of the quadratic equation. Only after the topic "Parabola" when its passed completely, they start studying the topic "Quadratic Equations". The solution of a quadratic equation is found by p-q-Formel, which implies reducing an equation from the form: $ax^2 + bx + c = 0$ to the form: $x^2 + px + q = 0$.

But in this case very often the calculations themselves become very cumbersome and inconvenient, because the coefficients of the equation, for example: $2x^2 - 5x + 3 = 0$ become fractional form reduced to the equation $x^2 - \frac{5}{2}x + \frac{3}{2} = 0$, where $p = -\frac{5}{2}; \quad q = \frac{3}{2}$. And p-q-Formel has the form: $x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(-\frac{p}{2}\right)^2 - q}$.

The formula for solving an equation through the discriminant is practically not used, although it exists, and it’s paid no significant attention to such an important topic as Viet’s theorem.

It can be emphasized that another significant drawback is the absence of the topic: "Solving quadratic inequalities", which would be a logical conclusion of the topic "Parabola and its equation", where are solved inequalities of the form: $ax^2 + bx + c \geq 0$ or $ax^2 + bx + c \leq 0$.

It should be noted that the topic "Linear inequalities" was presented in the 7th grade textbook, and basic knowledge for studying a topic "Parabola and its equation" is available. Despite this fact, we can see some tasks. Show with one of example:

"Betrachte die Gleichung $x^2+kx+20=0$. Bestimme für welche Werte aus R die Losungsmenge nicht leer ist". Let us briefly show the problem and their solution: $D=k^2-80\geq 0$.

The inequality $k^2-80\geq 0$ can be solved either by the method of intervals by writing: $(k+4\sqrt{5})(k-4\sqrt{5})\geq 0$ or consider that $y=k^2-80$ - is the equation of a parabola, find the zeros of the function (Nullstelle), $k^2-80=0;\Rightarrow k=\pm 4\sqrt{5}$. Schematically draw the graph of the function $y=k^2-80$ and in both cases get the answer: $x\in(-\infty;-4\sqrt{5})\cup(4\sqrt{5};+\infty)$.

One of the positive aspects is including the topics section: "Using quadratic equations to solve optimization problems" (Optimierungsprobleme), where, as is usually in German mathematics textbooks, most of the problems are of an applied nature. Here is one of them: „*Ein dreieckiges Grundstück liegt an einer rechtwinkligen Straßenkreuzung, es ist an den beiden Straßen 1, 20 m. bzw. 80 m. lang. Auf dem Grundstuck soll eine rechteckige Lagerhalle mit einer möglichst großen Grundflache gebaut werden. Die Seiten der Lagerhalle sollen parallel zu den Straßen verlaufen"* [316]. In fact, in this problem it is necessary to maximize the area of a triangle, but the condition is a real situation: it is necessary to find the maximum area of a land plot. Such kind of topics are absent in Ukrainian math textbooks.

Let's move on to the study of mathematics in grade 10th. Let's analyze in more detail the presentation of the material of the topic "Exponential function" (Exponentialfunktion) and "Logarithms" (Logarithmen). This topic begins with the study of its section: "Linear and exponential growth" (Lineares und exponentielles Wachstum), where pupils should solve standard problems, such as the classical problem of dividing the number of bacteria, to understand how to determine the number of bacteria that divide at a constant rate per unit of time, over a given time interval. This section also contains textual problems, for the solution of which it is necessary to use the formula of percentage problem.

It should be noted that the formula of percentage problem $A = A_0(1 + \frac{p}{100})^n$ is not used only to banking problems on profit, but there are many tasks on expenses, constant cheapening of goods, tasks of weight loss calculation for a certain period of time.

In addition, there are many applied tasks related to other areas of life. For example: “Vor einer Operation wird einem Patienten ein Narkosemittel mit einer Konzentration von 2 mg pro Liter Blut im Körper verabreicht. Die Konzentration im Blut verringert sich alle 10 um 5%. Weise rechnerisch nach: Die Konzentration im Blut sinkt um ca. 26,5% pro Stunde“. In this assignment, you need to track the percentage decrease in anesthesia concentration in the blood during time [317].

Thus, as it was emphasized above, the presence of a significant number of applied text problems is a huge positive factor of German mathematics. Thus, the question: "What is it for and where is it used?" immediately disappears. The applicability of the knowledge is obvious.

But on the other hand, the proposed model of learning-mathematical modeling of the posed problem is not always successful, because pupils do not yet have the experience of the researcher and do not have enough basic knowledge on which they could rely.

It would be more logical to give a set of mathematically sound theoretical knowledge in many topics, at least minimal experience in solving examples, and only then move on to problems. Since problems are a more difficult stage in mathematics, it requires more theoretical knowledge, an understanding of the mathematical apparatus that can mathematically describe it, and the ability to solve examples.

The concept of "Exponential Function" is introduced only after solving a large number of such kind problems. The function itself and its properties are discussed very shortly. Only one page of the textbook is devoted to the topic "Logarithms" (Logarithmen). The proposed tasks themselves are elementary.

Here are examples of the most difficult level, which exists in German textbooks: “Entscheide, welche der folgenden Gleichungen korrekt sind” (it’s necessary to determine which of the expressions are correct): $\log_3(9 \cdot 27) = \log_3 9 \cdot \log_3 27$. Or “Ordne

die Logarithmen nach ihrer Große ohne Taschenrechner" (its necessary to order without calculator the logarithmic): $\log_2 64;\ \log_3 64;\ \log_8 64$.

Note that neither are discussed in detail the properties of logarithms; it isn't any full information about their domain or range. There are no assignments on exponential and logarithmic equations.

And for comparison, the Ukrainian school textbook offers the following tasks as intermediate level tasks: $\frac{5}{3^x-6}+\frac{5}{3^x+6}=2$.

Here its necessary to solve equation, Or, other example, it's necessary to simplify: $\log_3 \cos^2\frac{\pi}{9}\cdot\log_{\cos\frac{\pi}{9}} 9$.

There are a tasks of a more complex level: it's given an equation for solving: $4^{x+\frac{1}{2}}+\frac{2}{4^x}+14=9(2^x+\frac{1}{2^x})$, or task about graph function constructing: $y=\log_{\frac{1}{3}}(x-2)\cdot\log_{x-2}\frac{1}{3}$ [318].

In the part of this topic, or earlier, it would be good to give the concept of progression (both arithmetic and geometric), since the solution of classical problems on the growth of the number of bacteria is described not through the exponential function, but through the geometric progression: $b_n=b_0\cdot q^{n-1}$ in many math textbooks, where b_0- the initial number of bacteria; q- the coefficient of their division, b_n - the number of bacteria for the n-th period of time.

Moreover, it should be taken into account that knowledge of progression is necessary to have when the problem asks to find the sum of n-elements, which occurs in an increasing sequence. Let us focus on one of the important dual topics of algebra and geometry "Trigonometry" (Trigonometrie).

Thus, the basis of trigonometry is studied in 9th grade; it is right triangle, the relationship between its sides, Pythagoras' theorem (Satz der Pythagoras). Moreover, the Pythagoras theorem itself, its formulation, is given only as a statement, no more than that; the calculation tasks are primitive.

But, let's return to the topic "Trigonometry", which is studied in 10th grade, and look at the presentation of the material in this case. Pupils are familiarized with the concept of periodic function, and the unit circle. They learn to determine the position of a point in the unit circle, through the ratio of the corresponding sides of a right triangle embedded in this circle.

The material is structured in such a way that pupils don't connect such concepts as trigonometric functions of an acute angle and right triangle. It is absolutely unclear to them what means originally these relations between the sides. And that the value of these functions depends only on the degree of the angle, but not on the dimensions of the sides of triangle [317,318].

There aren't given such important formulas as the basic trigonometric identity: $\sin^2 x+\cos^2 x=1$, relationships between trigonometric functions: $\tan x=\frac{\sin x}{\cos x}$; $\cot x=\frac{\cos x}{\sin x}$.

There aren't formulas of the form: $\sin(90^0-\alpha)=\cos\alpha$; $\cos(90^0-\alpha)=\sin\alpha$, and the formulas like: $\tan^2\alpha+1=\frac{1}{\cos^2\alpha}$; $\cot^2\alpha+1=\frac{1}{\sin^2\alpha}$.

As a consequence, pupils do not know about transformations of the form: $\sin(\frac{\pi}{2}\pm\alpha)$; $\cos(\frac{\pi}{2}\pm\alpha)$; $\cot(\frac{\pi}{2}\pm\alpha)$; $\tan(\frac{\pi}{2}\pm\alpha)$.

It should be noted that the formula: $\sin^2 x+\cos^2 x=1$ occurred once in a 10th grade textbook, but as just a only one task from many, whose condition was to prove that the given identity was true: $\sin^2 x+\cos^2 x=1$.

There is absolutely no clear connection between radians and degree measure. There are no tasks for converting the degree into radians and vice and back. Moreover, it's paid not enough attention to negative and positive angles, to the signs of trigonometric functions.

And this is one from the important factors. It is also striking that most German pupils are not taught to calculate without a calculator, so they often do not understand

why they got such an answer.

Thus, to the statement that $\sin 45^0 = \frac{\sqrt{2}}{2}$ the majority of schoolchildren reacted with the following phrase: "No, that's not what my calculator shows". Only after they recalculated the value of the expression: $\frac{\sqrt{2}}{2} \approx 0,707$, and convinced that $\sin 45^0 = \frac{\sqrt{2}}{2}$ and is $\sin 45^0 = 0,707$ the same, they agreed with this statement.

A table of values of trigonometric functions of some angles doesn't display In the course of the topic "Trigonometry", but it's familiar to Ukrainian schoolchildren, namely the values of sine, tangent and cotangent at $\alpha = 0^0;\ \ 30^0;\ \ 45^0;\ \ 60^0;\ \ 90^0;\ 180^0$.

The schoolchildren haven't any idea what are the Brady's tables, and that there are such tables, by which you can see the values of any angle, if you do not have a calculator at hand. The fact that one degree is 60 min isn't studied. For example, the tasks of the form: "calculate $\sin 45^0 20'$" hasn't been met once. The textbook doesn't pay enough attention to the signs of trigonometric functions in quarters of the Cartesian coordinate system, it becomes difficult to perceive the fact that $\sin(-45^0) = -\frac{\sqrt{2}}{2}$.

And the fact that $\sin 315^0 = \sin(-45^0) = -\frac{\sqrt{2}}{2}$ is difficult to understand for pupils.

The schoolchildren calculate the values of trigonometric functions as a fact, and do not ask the question "Why is it so?"

But, it's very important to understand the nature of this phenomenon, because the fact that $\sin(-x) = -\sin x;\ \ \cos(-x) = \cos x$ serves as the very basis for constructing the graphs of trigonometric functions, and recognizing their behaviour. In this case, the presentation of material in Ukrainian school is much better.

Already in the 7th grade in the course of geometry pupils study the right triangle, its properties, Pythagoras' theorem and as a consequence of this all the trigonometric function of the acute angle as the ratio of the corresponding cathetes and hypotenuse.

Knowledge is consolidated by learning formulas such as the basic trigonometric identity: $\sin^2 x + \cos^2 x = 1$, ratios between trigonometric functions:

$\tan x = \frac{\sin x}{\cos x}$; $\cot x = \frac{\cos x}{\sin x}$, the following ratios: $\sin(90^0 - \alpha) = \cos\alpha$; $\cos(90^0 - \alpha) = \sin\alpha$.

There are learning formulas of the form: $\tan^2\alpha + 1 = \frac{1}{\cos^2\alpha}$ and $\cot^2\alpha + 1 = \frac{1}{\sin^2\alpha}$. The table of functions for some of angles is studied and given as a basis for calculations: $\alpha = 0^0$; 30^0; 45^0; 60^0; 90^0 .

And pupils operate and deal only with these concepts for quite a long time (trigonometric functions as a consequence of the relations between the sides of a right triangle) [319]. It should be noted that the teacher doesn't always require learning the proofs and conclusions, but they are all there. And the fact that nothing is given without a evidence base, which allows a good pupil to deal with the question "Why is it so?"

Return to topic: "Trigonometry" in the Ukrainian school also occurs as in the German school in the tenth year of study. Thus, the same topic (in the Ukrainian school) is quite large itself, and is divided into subsections, where a huge number of various tasks are given. Thus, in the section "Degree and radian measures of angle" pupils solve a lot of tasks, they are from converting angles from radians to degree and vice versa. For example, to convert to degrees the angles in radians: $\alpha = \frac{2\pi}{3}$; $\alpha = \frac{7}{6}\pi$

Or, for example, to calculate the values of trigonometric functions, when their angles are given in the different form: $3\sin\frac{\pi}{2} + 3\cos\frac{\pi}{6} - tg\frac{\pi}{3}$. Learn to determine the values of trigonometric functions, relying not only on the properties, but also on the signs of the functions in different quarters of the Cartesian coordinate system.

For example, find: $tg\alpha$; $\cos\alpha$; $ctg\alpha$, if: $\cos\alpha = \frac{5}{13}$; $\frac{3\pi}{2} < \alpha < 2\pi$ .They study how it's possible to solve different trigonometric problems by involving applications of various properties of trigonometric functions.

For example, simplify: $(1 + tg^2\alpha)\cdot\cos^2\alpha - 1$. Or other example: calculate:

$\cos\left(-\frac{\pi}{6}\right)\cdot\sin\left(-\frac{\pi}{6}\right)+tg\left(-\frac{\pi}{4}\right)$. Tasks on formulas for double angles, addition of trigonometric functions are studied in detail.

For example, there are separate subsections "Double argument formulas" in the trigonometry topic, where a sufficient number of tasks are focused on solving double angle formulas. Such as, for example, to simplify the expression: $\frac{\sin 2\alpha}{(\sin\alpha+\cos\alpha)^2-1}$.

There aren't such sections as "Trigonometric equations and inequalities" in the German trigonometry course, and this topic takes up about 20 pages in the 10th grade algebra textbook (Ukraine). Both equations and inequalities are of different complexity. Here are examples: solve the equation: $5\sin 2x+4\cos^2 x-8\cos x=0$, solve the inequality: $2\cos^2 x+\sin x-1<0$ [319]

The section "Graphs of trigonometric functions" is divided into many separated parts: function $y=\sin x$, its graph and properties; function $y=\cos x$, its graph and properties, functions $y=\tan x$; $y=\cot x$, their graphs and properties.

Where the material is studied in details, and given both simple and quite complex tasks. Show with some examples: find the area of definition of a function: $y=\sqrt{\sin x+1}$; find the largest and smallest value of the function: $y=3\cos 2x-4\sin 2x$. Thus, we can see a significant difference in the presentation of this topic for pupils in Ukraine and Germany.

We would like to emphasise that the tenth year of study is very important in a German school because the pupils should take the MSA (Mittelschulabschluss) maths exam, the final exam, which is taken into account both when moving on to a higher level of education (grades 11-12 or 11-13, which depends on the state) and if the pupil wants to leave school and study in the college (Berufsausbildung). It should be noted that this exam is taken in school, but this simplifies the situation only visually, because there isn't any concept of "to help student or pupil" and giving him a grade higher as he/she got really, and in 99% everyone gets what he became.

MSA tasks aren't difficult, they cover the level of knowledge from 6th to 10th grades, and they are mostly situational textual tasks. It should be noted that due to the

autonomy of the regions of state lands and the content of the tasks in the exam in terms of complexity also varies.

There are two ways after the end of the 10th grade and all depends on the pupil's performance and desire. On the first one, he/she can go to study a profession (Berufsausbildung), which is practically college model in Germany and study to become a nurse, accountant, bank employee, and so on. In this case, secondary school education in Germany is considered to be completed.

On the second one, it's possible to go on to the Oberstufe level, to study within the school system for two or three more years and then to continue their studying at university. Ukrainian pupil can leave school after 9th grade and go to college to study a profession, but in this case he/she completes the missing years of the same school programme in college. Or he/she continues his/her studies until he/she completes school and receives a certificate of secondary education after the 11th grade.

Let's make a general conclusion about German secondary mathematics education as one of the examples of European education. Thus, for choosing to study in another country, it is necessary to carefully consider the disadvantages and advantages that can be obtained in this case. The factor of staying in a more economically successful country does not guarantee the quality of education at all.

It should be taken into account that can be huge differences into the level of training, for example, it is Germany, where significant difference in training between the regions of states exists (Länder).

It should be borne in mind that the older the prospective pupil, the higher and stricter the language requirements are, and this should be prepared for from the outset. In many cases, such a change of language environments can lead to the loss of several years, even to the effect of psychological distress and non-acceptance of the situation.

In addition, in case of moving from a stronger environment to a weaker one, the demotivation effect is often triggered, according to the principle "I am already the best", why else should I work? The general analysis of the teaching methodology and the content of the material in the course of German mathematics in 5th -10th grades have shown the following: an unconditional factor that can be attributed to the positive is

the very large presence of textual tasks of an applied nature. They force to think, to model, to search for solutions.

But there are a lot of negative aspects that should be taken into account, such as: chaotic presentation of material, lack of rigorous proofs and theorems, clear formulations, lack of sufficient number of examples. From our point of view, this model of mathematics can be considered more as an introductory as fundamental training.

The situation changes at the stage of education in 11^{th} -12^{th} (13^{th}) grades. As mentioned above, often the models of education are different from their structure in many countries. Such a "superstructure" as Oberstufe and preparation for the Abitur exam doesn't exist in the secondary education system in the post-Soviet space.

The structure of the Abitur exam is radically different from the independent maths exam in Ukraine, and it is more likely that a Ukrainian excellent student will fail this exam than pass it [320].

Thus, in Ukraine, independent maths exams cover the whole course of school maths, and they may contain quite easy tasks of the 6^{th} -7^{th} grade level. Such as: "reduce a fraction, simplify expressions". In addition, some of the tasks are given with answers; there is just a possibility to guess the correct answer.

Th Abitur exam is aimed only at assessing and determining the level of knowledge of applicants who want to get a university education in the future, it doesn't include tasks of the 6th-7th grade level, where pupils must demonstrate his/her ability to reduce fractions. This level of knowledge has already been tested at the end of 10^{th} grade when students take the MSA (Mittelschulabschluss) exam. The tasks are based only on the knowledge of the Oberstufe (10-13 years of schooling), the exam doesn't test part and it isn't possible to pass the Abitur in mathematics without having sufficient knowledge of this level, a good knowledge of secondary school mathematics will not help.

This structure explains the "non-recognition" paper of secondary education certificates of other countries, and the requirement to study in Germany for at least a one year before entering university, because level the knowledge of mathematics at the

Oberstufe often corresponds to the level of mathematics of 1-2 courses of Ukrainian universities. Besides, there is a purely formal factor and in Ukraine the secondary education is a full course of study at school. In Germany, it is only 10 years of study, and to enter a German university is required the education at Oberstufe. And that is the reason why German universities require from applicant of outside country additional training at the Studienkolleg very often.

At the same time, the level of material and its presentation, which is studied by German pupils in 11-12, sometimes at 11-13 grades is very different from what is studied by Ukrainian pupils. The attitude towards pupils at Oberstufe is different.

They are pupils, but look like students in the school. They aren't required to attend school (Schulpflicht) and are freer in their action. Let's take a closer look at some of the topics and their methodological presentation at the Oberstufe.

The construction of the courses is such that they are oriented in genera l on the preparation for university studies, and those topics that are continued in university courses such as "Mathematical Analysis", "Vector Algebra", "Analytical Geometry", "Probability Theory" are considered in more detail and in full. The same topics that don't have such a "university" orientation, on the contrary, are studied in a more simplified and shortened form.

For example, in the process of geometry studying, considerable attention is paid to vectors and operations with them. When the Ukrainian pupils are limited with deal to make the linear operations with vectors in algebraic and geometric form, and to know how to calculate the scalar product, here there is a whole course, which is called: "Kurse für Analytische Geometrie (Course of Analytical Geometry), in which there is a subcourse "Vectors and Straight Lines" (Kurse für Vectoren und Geraden).

Pupils study in detail the scalar and vector products learn the formulas for finding the area of a triangle and parallelogram through the modulus of the vector product of vectors. In addition, despite the fact that the concept of mixed product of vectors is not introduced, pupils know how to find the volume of a pyramid through the ready formula of the mixed product.

It should be emphasised again that it is necessary to take into account the constant difference in definition, formulations and concepts. Thus, if a Ukrainian pupil (student) is used to see the coordinates of a vector in the form of a matrix-row record, namely $\vec{a} = (a_x; a_y; a_z)$, but in German textbooks they record is in the form of a matrix-column and it is very often confusing very often: $\overline{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.

There are the demands to the set of basic knowledge of matrix calculus very high through the fact that the vector is linked to a matrix calculus.

Pupils know types of matrices, know how to find an element of a matrix, can add and multiply matrices, find an inverse matrix, moreover, there are a number of tasks to find eigennumbers and eigenvectors of a matrix.

As an example, we can show a task where it is necessary to find eigennumbers for a square matrix of the third order:

Berechne die Eigehwerte der Matrix $A = \begin{pmatrix} -3 & 4 & -2 \\ -6 & 7 & -3 \\ -6 & 6 & -2 \end{pmatrix}$.

There is not a lot of such kind of tasks, the level of complexity of disclosure of any topic depends on the state of Germany, but nevertheless such assignments exist.

The course of "Analytical Geometry" deserves special attention. There isn't such course of Analytical Geometry, as well as the course "Matrices" at all within the framework of school in Ukraine. The Ukrainian pupil know only two types of straight lines in the plane: the general equation of a straight line: $ax + by + c = 0$, and the equation of a line with a slope: $y = kx + b$.

The plane exists only as a concept, and the equation of the plane is not given at all, not to mention such sections of the course as "Plane and straight line in space".

At this level, even a qualified maths teacher may have difficulty understanding the material.

It is in upper level of the school is possible to have a deal not only with a new topic, but also with completely different method of presenting and explaining the

material. Here is an example of one of the tasks. It is necessary to determine which of the points lies on the given line:

Welcher der Punkte liegt auf der Geraden $X = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 18 \\ -3 \\ -6 \end{pmatrix}$.

The following answer options are offered:

1) $P(5; -1; 3)$; 2) $P(17; 2; -3)$; 3) $S(11; -2; -3)$.

Due to the unusual writing of the task, most of the Ukrainian students (this topic isn't studied in Ukrainian school) face difficulties in solving this problem.

Let us show the scheme of its solution. Thus, in this case the straight line is written in parametric form in the form:

$$X = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} + \lambda \begin{pmatrix} m \\ p \\ s \end{pmatrix}.$$

For a Ukrainian student (this material is studied in Ukraine in the first years of university studies) it is more usual to write the following form:

$$\begin{cases} x = x_0 + \lambda m \\ y = y_0 + \lambda p \\ z = z_0 + \lambda s \end{cases}.$$

Further, the native student should remember that the parametric equation of a straight line in space is derived from the canonical equation:

$$\frac{x - x_0}{m} = \frac{y - y_0}{p} = \frac{z - z_0}{s} = \lambda.$$

Let us show that the point S is the answer to this problem. Based on the condition, we have:

$$X = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 18 \\ -3 \\ -6 \end{pmatrix}.$$

$$\frac{-1 - x_0}{18} = \frac{0 - y_0}{-3} = \frac{1 - z_0}{-6} = \lambda.$$

Let's substitute the coordinates of the point $S\,(11;-2;-3)$ into the equation and got:

$$\frac{-1-11}{18}=\frac{0+2}{-3}=\frac{1+3}{-6}=\lambda\,.$$

Whence we obtain that λ everywhere is the same and equal to $\lambda=\frac{2}{3}$.

Thus, the point S is a point that belongs to the given line.

Here is another example. The equation of the plane in German mathematics is also given in parametric form (Ebenengleichung in Parameterform) and its form has the following notation:

$$X=\begin{pmatrix}1\\3\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\-3\end{pmatrix}+\mu\begin{pmatrix}-4\\1\\1\end{pmatrix}.$$

And here is the equation of the plane in general form: $Ax+By+Cz+D=0$ it is called the equation of the plane in normal form or “Ebenengleichung in Normalenform”, although in our course (in Ukraine) the normal equation of the plane is called a completely different equation [321].

In the German maths course at Oberstufe, it’s paid a great deal of attention to the functions study. Thus, at 12th grade of maths course (Bavaria region), pupils learn the theory of limits, learn all kinds of asymptotes, and know how to construct and investigate quite complex functions.

Show with task: Geben Sie $\lim\limits_{x\to+\infty}\left(\frac{1}{x}-3\right)$ und $\lim\limits_{x\to-\infty}\left(-\frac{1}{x}-3\right)$ an und begründen Sie. Thus, students are asked to find left-hand and right-hand limits, and explain the obtained answer. As mentioned, great importance is given to the study of functions, and many of the activities are different from what students are used in Ukraine.

For example, on the topic: "Fractional Rational Functions" (Gerochen-rationale Funktionen), students find all kinds of asymptotes (this material is studied in the first year of university). Here is an example of finding an oblique asymptote (Schrage Asymptote).

It is necessary to find the oblique asymptote for the graph of a function:

$$y = \frac{3x^2 + 5x + 3}{2x}.$$

In Ukrainian universities, students are taught that the oblique asymptote can be found in the form of the equation of a straight line: $y = kx + b$, where $k = \lim\limits_{x \to \pm\infty} \frac{f(x)}{x}$.

If $k \neq 0; k \neq \pm\infty$, then they look for: $b = \lim\limits_{x \to \pm\infty} (f(x) - kx)$. Let's shown the necessary calculations and get:

$$k = \lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{3x^2 + 5x + 3}{2x \cdot x} = \frac{3}{2},$$

then we can find the value of the coefficient:

$$b = \lim_{x \to \pm\infty} (f(x) - kx) = \lim_{x \to \pm\infty} (\frac{3x^2 + 5x + 3}{2x} - \frac{3}{2} \cdot x) = \lim_{x \to \pm\infty} (\frac{3x^2 + 5x + 3 - 3x^2}{2x}) = \frac{5}{2}.$$

Substitute all into the equation of the straight line $y = kx + b$, we obtain the equation of the inclined asymptote:

$$y = \frac{3}{2}x + 2,5$$

German pupils use another way. They only look for the limit of the function $f(x)$ itself when $x \to \pm\infty$ at and get:

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{3x^2 + 5x + 3}{2x} = \lim_{x \to \pm\infty} \left(\frac{3x^2}{2x} + \frac{5x}{2x} + \frac{3}{2x} \right) = \frac{3}{2}x + 1,5 .$$

And the part that "exists" after finding the limit is the equation of the oblique asymptote. Namely: $y = \frac{3}{2}x + 2,5$.

It should be noted the "love" of German mathematics for the construction and study of all kinds of graphs, to which whole sections of the textbook at the Oberstufe level are devoted.

For example, one of the tasks that occur throughout the unit is to carry out a complete and complete investigation and construction of the function and construct their graphs:

$$f(x)=\frac{5x+1}{x};\quad f(x)=x+1+\frac{1}{x+1};\quad f(x)=\frac{x^2-4}{x^2}\ .$$

It should also be noted that much more importance is given in German gymnasium classes in contrast to Ukrainian schools to probability theory and mathematical statistics (Stochastik, Wahrscheinlichkeit, Statistik). The first tasks on this section are found already in the 8th grade, but as it was shown above, they are more tasks for gaining knowledge of an overview character: the ability to construct a diagram, calculate the simplest combinatorial problem, or calculate the simplest probability, according to the classical formula.

But it is a whole section of mathematics at the Oberstufe level, and also in the Abitur exam, a part of the exam is devoted to this type of tasks [322].

Here the presentation of material is very different from that what is usual in Ukraine as part of the university curriculum; this part of mathematics is not present in school at all.

A significant disadvantage that complicates the perception of the topic is the practical absence of such an important subsection as "elements of combinatory" and the immediate transition to the calculation of probabilities. Moreover, attention is not focused on sufficient understanding of such concepts as complementary, simple, compound, sure, impossible, mutually exclusive events, etc.

The definitions themselves are joint to set theory, but it is also not given completely enough, and the pupils have the difficulties with questions such as "Are such concepts as $P(A\cap B)$ and $P_A B$ are identical or not?"

There are a lot of tasks for building trees of events (Baumdiagramme), tasks where probabilities are given in a certain form, in the form of tables (Vierfeldertafeln), where in the cells at the intersection of events is the probability of their conditional appearance. This way of tasks assignment is standard for German mathematics and is absolutely not usual for Ukrainian students.

Thus, let us show how the dependence and independence of events are explained to German pupils. Thus, at the beginning of topic, when pupils start studying the

dependence of events (Bedinge Wahrscheinlichkeit), and the course starts with dependent events, tasks are given in the form of tables (Die Vierfeldertabelle).

It is an example of such kind of the task. There is the table, where the number of female and male cats with red and not red hair and their total number are given (Table 1). Such presentation of material isn't usual for Ukrainian pupils (students). For example, the element $a_{11} = 3$ shows the number of female cats with red hair.

Table 1. Number of female cats and male cats, depending on hair colour

	Katze	Kater	gesamt
Rote Fellfarbe	3	17	20
Andere Fellfarbe	19	10	29
Gesamt	22	27	49

One of the questions in this task is: "Determine the probability that an animal with red hair is taken, provided that it is a female (male) cat". (Bestimmen Sie die Wahrscheinlichkeit, dass das Tier rotes Fell hat unter der Bedingung, dass es weiblich (männlich) ist.

The calculations would be as follows: $P(A) = \frac{17}{20}$ -probability that the animal is a male cat;

$P(B) = \frac{3}{22}$ probability that the animal is a female cat;

check this answer: $P(A) + P(B) = \frac{17}{20} + \frac{3}{27} = 1$.

These two events are mutually exclusive; the sum of probabilities is equal to one.

Now let's see how the independence of events is explained. Here is an example. According to the given table with events, it is necessary to determine whether the events are stochastically independent or not (Table 2).

Let us explain the contents of the table. As it is obvious that in the last column and the bottom row is the sum of opposite events:

$$P(Q) + P(\overline{Q}) = 1 \text{ and } P(S) + P(\overline{S}) = 1.$$

At the intersections stands the probability of their joint appearance of events ($P(S \cdot Q) = 0,245$ - probability of joint appearance of the events).

Table 2. Probabilities of appearance of events P and Q

	S	$\overline{S}$	
Q	0,245	0,455	0,7
$\overline{Q}$	0,105	0,195	0,3
	0,35	0,65	1

The solution of this problem is quite easy, if we understand the structure of the condition of the problem itself. Thus, let us find the following probabilities: $P(S \cdot Q) = 0,245$ and $P(S)P(Q) = 0,35 \cdot 0,7 = 0,245$.

In this case, since the equality is fulfilled: $P(S \cdot Q) = P(S) \cdot P(Q)$, and hence these events are stochastically independent. Note that stochastic independence, in our case, is nothing more as the theorem of multiplication independent events.

Thus, the methodology of presenting the material differs greatly, and not always in a better and more optimal way. And without proper training it would be difficult to understand exactly such explanations of "independence", "dependence", "complementarities" of events.

There are studied the absolute probability (absolute Häufigkeit), Bayes' formula (Die Regel von Bayes), elements of the theory of correlation and regression (Korrelation und Kausalität) at this course. But they are studied in the first year of University in Ukrainian higher education institutions.

Thus, we can say that it is impossible to give an unambiguous answer to the question: "Where is the best education?", as result of analysis of mathematics teaching at the secondary (Gymnasium) and higher level (Oberstufe).

But the statement that education in the post-Soviet space is much worse isn’t true. The structure of education is very different, its content and implementation in the time interval are different.

And therefore all these factors should be known and taken into account when somebody choose to study in another country. It is necessary to clearly understand your capabilities and the order of actions at each stage of the choice. If you consider studying abroad or find the emigration as a goal of getting a "very good education", what is more difficult and see it as something immutable, then in most cases it can lead to disappointment.

First of all, it is necessary not only to study the country and its peculiarities, but also to be very well prepared in terms of language, and to take into account that it can be the cardinal difference in preparation between the regions (Länder) in some country, why they (regions) are quite autonomous in some of countries.

Thus, as it was shown, when the secondary schools in the post-Soviet countries have more of the opportunities to function well, namely: there are enough qualified teachers, there is a good technical base, classes are not overcrowded., then they (the school) are much better in terms of organization, equality of opportunities (there is no clear division of school-children into successful and lagging behind, etc.) for pupils.

As for further education, it is necessary to take into account the absolute multifactorial nature of this process. Namely: the financial possibilities of the family itself, to take into account the structure of education, language training, and the peculiarities of the country.

As mentioned above, in some German regions, to have a school education diploma (from other country) doesn’t guarantee immediately admission to university, because the German education system has Oberstufe as the "layer" between school and university.

And studying at a university, especially if the faculty is complex, then it requires some level of knowledge that is given exactly on these two or three years of study at Oberstufe, and not only knowledge as a set of information, but also consideration of the methodology of its presentation.

But in this case the key to successful study is level of preparation, which must be good enough. Very good results are obtained by a model that is used among students

very often now: to get a bachelor's degree in homeland and entering a master's program in another country. In this case, the risks of failure are minimal.

But in any case, the questions of international education are relevant questions of our time, because the world has entered the era of globalization. Despite the fact that it is important and relevant, there is no any solution answer to the question "What is the best solution?" And each case requires careful study and consideration.

The next level of education after passing the Abitur exam is university. Let us briefly consider this level of education. Again, there is an opinion that higher education in Ukraine is quite bad, and not comparable in quality with European and world education.

Look at this problem as a whole. First of all, it is necessary to find out where such information comes from. There was a constant information that there are a lot of educational institutions in Ukraine, and their number is quite large in comparison with the world number. Such kind of information was in the years before the war.

But according to statistics, the proportions between of number of higher education institutions and the number of population in Ukraine is approximately about 1:6; in the USA 1:19; in Germany the ratio is 1:4. Thus, the number of higher education institutions in Ukraine is not critically high by the world standards.

Now let us pay attention to the fact who compiles the world university rankings. There are three main rankings.

They are Times Higher Education University Ranking (THE), World University Rankings (QR) and Shanghai Ranking (ARWU). The author of the first ranking is Times Higher Education magazine, based in London. The author of the second rating is also Times Higher Education magazine in cooperation with other corporations. The author of the ARWU rating is Shanghai University and it was founded only in 2020. According to these rankings, the first positions are occupied by English-speaking universities in the US and UK, thus promoting themselves.

Of course, it would be unwise to compare higher education opportunities in Ukraine and advanced universities in the United States. But is everything so bad?

So, education in the modern world, especially higher education, is a business

that brings good dividends, not only in the form of money. Developed countries earn millions of dollars on the sale of educational services and work in this direction, so, for example, the United States sold approximately about 30% of all their educational services [323].

Besides the profit, in many universities higher education costs prohibitive money, it's given a lot of hidden effects: it attracts the most talented and capable young people; it creates the effect of "forming an employee for himself", it develops of research centers and scientific schools on the basis of universities, it's made popular the language and culture of the country of study, and as a consequence, it's made strengthened the role of the country in the world arena, and much more.

In addition, due to the fact that the "client" (it is the student in this case) is in the country of study for a long time, there is an investment in other aspects of life (medicine, recreation, shopping, payment for accommodation, etc.).

Show some more interesting facts. For example, one of the main criteria for choosing to study at Harvard University (42%) is its prestige and about 25% of students are foreigners [324].

Therefore, the hidden goal of each country in the field of education is to promote itself, to suppress the competitor, to make the conditions of study more attractive, realizing that these will be followed by direct monetary investments. And further, the client who has received a service in the form of a diploma and luggage of knowledge should promote on the labor market himself.

For example, the significant increasing the number of foreign students at German universities in 2023 has led to the possibility of introducing additional financial charges in addition to direct payment, and the increasing of the number of foreigners serves as an excellent promotion for the university itself [325]. Interesting is the fact that the independence of educational institutions in the U.S. is so developed that it is quite easy to get the status of a university. As result the status of college and university is quite blurred, so you really need to be careful about the choice of educational institution, because not every degree opens the door to success.

The situation is the same in many educational institutions in Germany.

So, Ludwig-Maximillian's-University of Munich will provide excellent training, but the requirements will be quite strict not only for admission, but also for studying in it. If an applicant from another country can enter study and successfully graduate from this university, its diploma will open many doors.

But there is often a situation when somebody studies only for foreign diploma, for the sake of prestige. A lot of money and effort are invested, and the situation may turn out to be far from imaginary. Thus, according to their structure, world universities can be divided into two categories: those that provide real knowledge (but it is really difficult to study in them and necessary to it all time), and those that only promise a lot.

But if you have a diploma, even in the most rich and successful country, but your profession isn't in demand, you don't have the necessary luggage of knowledge, and in addition your university doesn't stay in the list of prestigious universities, you also risk to remain in the list of unemployed.

Therefore, it is important for a person who has decided to get education not in the home country to find out the following points. If we are talking about Germany, then:

- whether he/she is ready to compete with students who, for example, have the Bavarian Abitur level, if we are talking about the University of Munich or university of this level;

- whether he/she is sufficient for understanding the material presented, and the level of language is enough for this;

- whether he/she is prepared to understand the material based on the fact that in the system of secondary education is different. There was a different presentation of the material, some definitions and formulations, etc.;

- whether his/her profession will be competitive in the labor market when he/she graduate;

- whether he/she is able and ready to compete;

- whether he/she has the strength, opportunities and financial resources, which is important to start and finish education.

Let's take as an example, education at the Faculty of Economics of private University of Applied Science (Germany). The costs of education are from 10.000 Euros per year at this time. [326]. It's necessary to add the costs of flat, food and living expenses to it. But the level of knowledge is not very high, as we can see the list of requirements for, but the existence is one of important item, it stays on the website of this university. Let's see what is offered to study in the course "Business Mathematics", which is read for students in the first year of study. So, students should repeat the rules of actions with logarithms, which are studied in detail within the 10th grade of Ukrainian school by the topic "Functions" (Figure 1).

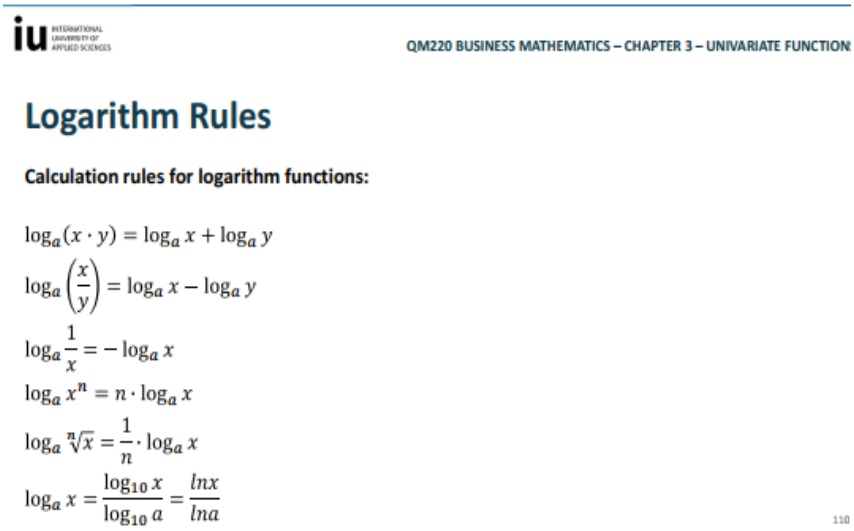

Figure 1. Logarithm Rules

Next, let us pay attention to a very interesting fact. In the same topic it is necessary to find the zeros of the function: $y = x^3 - x^2 - 8x + 12$.

That is, the points where $y = 0$ or $x^3 - x^2 - 8x + 12 = 0$. It is an equation of the third degree, it is taught in the school course of mathematics that for a polynomial in the form: $y = x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_0$, where $a_1 = 1$.

In this case the roots of the polynomial (the root is the number that converts the polynomial to zero), will be the divisor of the free element a_0, if are exists and are an integer. In our case, the roots of the polynomial $x^3 - x^2 - 8x + 12$ must be found among the divisors of the number: $a_0 = 12$.

Let's see how this kind of material is given in the lecture (Fig. 2). The lecturer proposes to guess the value of the root! Although there is no any difficulty to find it.

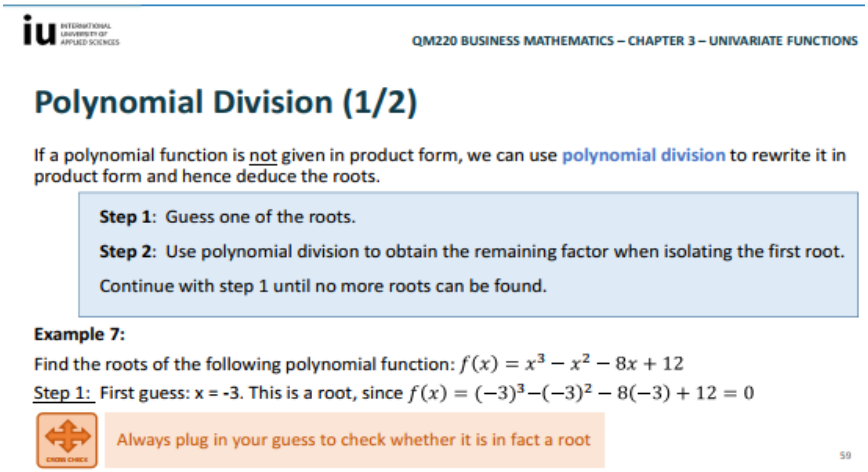

Fig. 2. Part of Lecture Polynomial Division

Schow the possible way of solution this problem. These are the numbers $x_1 = \pm 1;\ \pm 2;\ \pm 3;\ \pm 4;\ \pm 6;\ \pm 12$.

It is necessary to substitute all these numbers into polynomial, and see when the expression $x^3 - x^2 - 8x + 12$ turns to zero, therefore that numbers will be the root of the polynomial.

Sometimes for calculation is enough to find only one of roots numbers. So in our case it is $x = -3$. Then the second part of the theorem comes into force. If a number x_1 is a root of a polynomial $P_n(x)$, then polynomial is always possible to divide without any rest: $P_n(x) : (x - x_1)$. We obtain: $x^3 - x^2 - 8x + 12 : (x + 3) = (x^2 - 4x + 4)(x + 3) = (x - 2)^2 (x + 3)$.

And get: $y = x^3 - x^2 - 8x + 12 = (x - 2)^2 (x + 3)$.

Moreover, that can be that the rules for polynomial roots somebody finds seminars. But the next figure shows us the part of the seminar, where the value of roots is given as a fact too . (Fig.3)

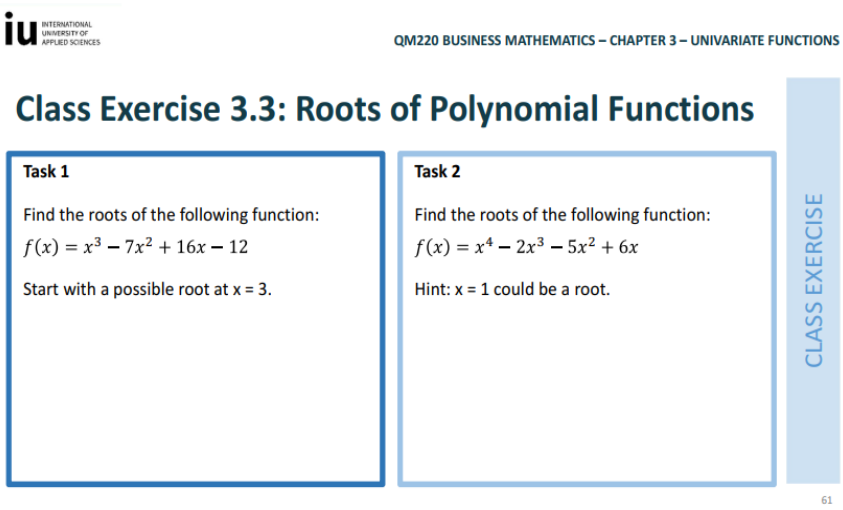

Fig 3. Part of Seminar Polynomial Division. Roots of Polynomial Function

There are given only simply elementary examples of tasks in the part of course "Differential Calculus", and they repeat the elementary level of school mathematics (Fig.4) .

.

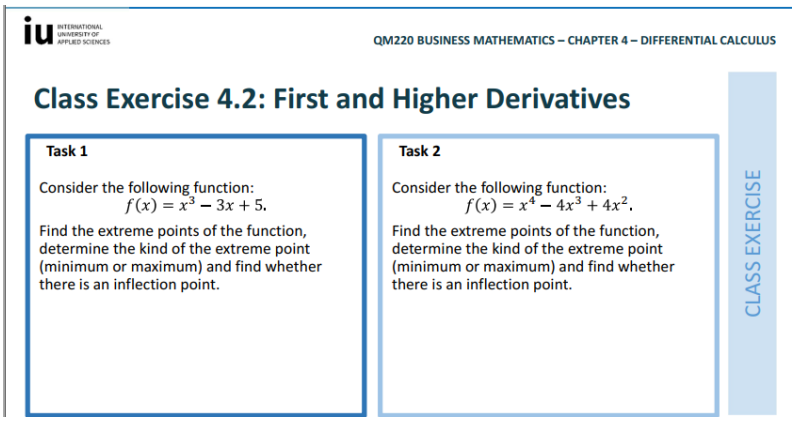

Fig. 3. Part of Seminar First and Higher Derivatives

There aren't any theorem, proof or axiom concepts in all course, definitions are emphasized no clear.

The section on "Probability Theory" contains only the basics of combinatorics. Perhaps the university offers a separate course "Probability Theory and Mathematical Statistics", but then it would be more logical to place the tasks of this section in the corresponding course. Otherwise, the connection is lost between the topics, since the elements of combinatorics are followed by the topic "Functions".

Let us compare the requirements for applicants of the University of Applied Science with the Ludwig-Maximilians-Universität München [327].

First, the university is free of charge. Second, the requirements for admission are: German level C-1, prior year of study at Studienkolleg, or a year of study at a national (home country) university.

These requirements are quite reasonable, as it was shown above the German school has a level of Oberstufe and therefore the level of the German applicant is higher than any Ukrainian, even with a good level of knowledge. Also would be intersting to analyze the reviews about studies. Applicants and students complain about high competition, difficulties in aplyment and hard studies.

In addition, it should be taken into account that Bavaria itself is the most expensive region to live in, and of course, accommodation will not be cheap. This is also noted in surveys of applicants and students. It should be noted that many state universities, which have a certain prestige, require foreigners to either study at the Stüdienkolleg, or a year of study at a university in their home country, or to finish studying at the Oberstufe of a German school.

It should take into account that the programs in Studienkolleg are also oriented to the some university, or to a group of universities. They are not equal all in the country or region. And in Stüdienkolleg it is still necessary to enter, very often education there, though not expensive, but paid.

Let's look at the level of tasks on mathematics for admission to the Stüdienkolleg of the University of Munich [328]. Thus, the applicant is offered to simplify such kind an expression:

$$\left(\frac{(-2b)^{2n}}{-(h^{2n}a)^{0}}\right)^{4} : \left(\frac{\frac{1}{2}(-h)^{2n+1}}{-2b^{2n}\cdot h}\right)^{-3}$$

Moreover, this task does not belong to the difficult ones, as here you just need to choose the correct answer from ten offered after your calculations.

Or an such kind of the task: you need to find a solution to a trigonometric equation for a given interval: “Bestimmen Sie alle Werte $x \in [0;2\pi[$ fur $2\sin^2 x - 3 = \cos x$ “.

In addition, there are lot of geometric tasks. It becomes clear that with a low level of schooling is not possible to enter without additional training.

At the Studienkolleg of the University of Coburg (also Bavaria region), the tasks are much simpler. For example, you can find very elementary tasks in the math entrance exam. Such as simplifying fractions: „Berechnen und vereinfachen Sie die folgenden Terme so weit wie moglich“: $\frac{d^2-c^2}{(c-d)^2}$ and $\frac{a^4}{c-3}:\frac{c^{-2}}{a^{-2}}$.

Thus, it can be seen that the world has a lot of quite serious educational institutions and “diploma printing factories” too . The given comparative analysis has shown that the choice of the "right" and necessary education is a very serious issue. And it is necessary to approach it not only with a desire to study in another country, but with all the seriousness of this choice. It is important to be able to compare all the features of learning and teaching, including the school system and higher level too.

If we talk about secondary education, it is necessary to understand when, at what stage it is better to join the system for minimizition of losses. It is also necessary to choose the right place of education, to choose an institution that will provide the highest quality secondary education, which will be a start to higher education.

It is necessary to understand if the secondary education document issued in the home country will be recognized as equivalent to the country of intended study or not. If doesn’t, it’s necessary to clearly understand what time and financial efforts should be taken for its recognition, how much additional study will be needed.

When choosing to study at a university, you should also know clearly that the high price of studies, loyalty of admission and promises of 99% employment aren’t t guarantees of success in the future. The cost of everyday life is one of the important factor, as free study in prestigious universities is often accompanied by a very expensive life in the region or country. Its nessecary to take in account that the structure of education itself is very different from what Ukrainian students are used to.

European students are more independent in their choice of courses and are responsible for their studies. This means that, when he/she has to pass the course "Higher Mathematics" during the first two years of study. The student must do this in

any way, despite lecturers or teachers the instructors may change during the time. Such kind of help that teachers provide in Ukraine, so lot of different indulgences that are not available here. This is why many studies are delayed for years: it is necessary to study and work and manage everything during one time and it’s really difficult.

In many cases, it is really better and easier to finish your education in your home country, remembering that there are a number of universities in your country, the diplomas of which are recognized on a global scale, and are valued higher than the diplomas of "average" European universities. And this is really so.

Or it is possible to use the model of completing a bachelor's degree at home and try to get a master's degree abroad. When choosing an institution, it is important to remember only one that education, whatever it is, is a business and to calculate your learning way to get the most out of it.