SECTION 12. MECHANICAL ENGINEERING

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12.1 Simulation of vibration resistance of a tool holder with dynamic vibration damper

Accuracy of high-speed cutting processes is largely limited by the level of vibration of the rotating tool holder, caused by the irreducible dynamic imbalance of the oscillating system [389, 390, 391]. Laboratory tests of a patented design of a dynamic vibration damper (DVD) based on a flywheel with a built-in ball friction multiplier (BFM) showed the effectiveness of controlling the amplitude-frequency characteristics (AFC) of the oscillatory system in a wide range of values of the mandrel rotation speed and the BFM gear ratio [392, 393]. The effect is due to the known quadratic function of the reduced moment of inertia of the flywheel on its rotation speed, as well as the dissipative properties of the flywheel. No less significant is the effect on the accuracy of high-speed grinding of the rigidity of the supports, spindle, tool wear, as well as the influence of gyroscopic forces [391].

The purpose of the study is to simulate forced vibrations of a tool holder with DVD in a wide range of angular velocities of the flywheel to verify the possibility of controlling the vibration resistance of the system.

As a calculation model, the oscillatory system "rotor-DVD" in the form of a rigid rotor on elastic supports A and B is considered (in Fig. 1). The beginning of the fixed coordinate system $X_0Y_0Z_0$ is taken to be point A_0 – the left support of the rotor in the equilibrium position. The beginning of the moving coordinate system $X_1Y_1Z_1$ is taken to be point C - the center of mass of the system (in the equilibrium position - point C_0). It is assumed that the coordinate system $CX_1Y_1Z_1$ moves translationally together with the center of mass of the "rotor-DVD" system relative to the stationary system $A_0X_0Y_0Z_0$. At point C there is a coordinate system $CXYZ$, rotating together with the rotor around this point (in Fig. 1).

The rigidities of the supports c_1, c_2, located at a distance L, are assumed to be the same in all directions of linear elastic deformations under the action of an external load in a plane parallel to the fixed system $X_0A_0Y_0$. The coordinates of the supports are taken as generalized coordinates: $x_A=x_1$, $y_A=y_1$; $x_B=x_2$, $y_B=y_2$. The Euler-Krylov angles are designated α and β. The projections of point A are the points A_x and A_y, the projections of point B are the points B_x and B_y. Line A_0B_0 corresponds to the position of elastic static equilibrium of the rotor axis. The center of mass of the system (point C_0) is shifted relative to the rotor axis by the amount of eccentricity e, the main axis of inertia of the rotor C_1Z_c forms an angle δ with the geometric axis C_0Z (axis of rotation, in Fig. 2).

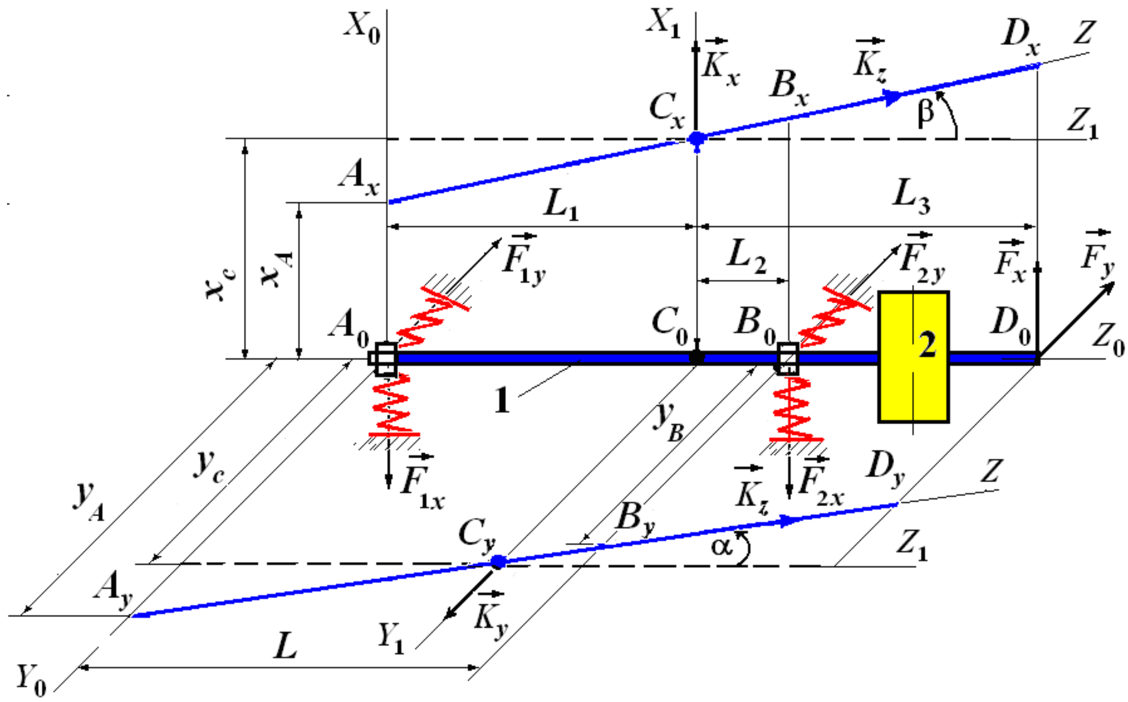


Figure 1. Design diagram of the oscillatory system (1 – rotor; 2 – DVD)

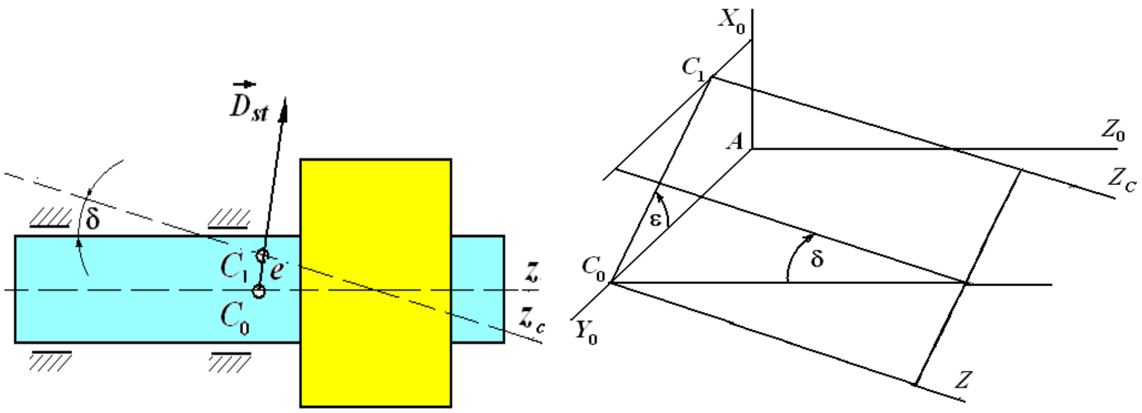


Figure 2. Scheme of dynamic imbalance of the “rotor-DVD” system

Static imbalance of the rotor is characterized by the value $D_{st} = me$, and dynamic (torque) imbalance is that the axes C_0Z and C_1Z_c do not intersect at point C_0 at $e \neq 0$. The planes of location of the C_0Z axis and the center of mass of the system, as well as the C_0Z and C_1Z_c axes, form a dihedral angle ε. Point D_0 corresponds to the installation location of the cutting tool and has projections D_x and D_y.

Oscillations along the C_0Z axis are not taken into account. Coordinates of the center of mass of the "rotor-DVD" system, taking into account eccentricity:

$$\left.\begin{aligned} x_c &= C_0C_x + e\sin\omega t = x_1 + L_1\sin\beta + e\sin\omega t = (L_2x_1 + L_1x_2)/L + e\sin\omega t; \\ y_c &= C_0C_y + e\cos\omega t = y_1 - L_1\sin\alpha + e\cos\omega t = (L_2y_1 + L_1y_2)/L + e\cos\omega t. \end{aligned}\right\}. \quad (1)$$

Angles α_1 and β_1 between the C_0Z axis and the projections of the main central axis of inertia of the rotor, respectively, on the planes $C_0Y_1X_1$ and $C_0X_1Z_1$:

$$\left.\begin{aligned} \alpha_1 &= \alpha + \delta\sin(\omega t - \varepsilon) = (y_1 - y_2)/L + \delta\sin(\omega t - \varepsilon); \\ \beta_1 &= \beta + \delta\cos(\omega t - \varepsilon) = (x_2 - x_1)/L + \delta\cos(\omega t - \varepsilon). \end{aligned}\right\}. \quad (2)$$

The external harmonic disturbance is given by the equation

$$F_x = F_0\sin\omega t; \;\; F_y = F_0\cos\omega t. \quad (3)$$

The moment of inertia of DVD relative to any axis perpendicular to A_0B_0 and passing through point C_0 is designated J_e. Projections of kinetic moment on the X, Y, Z axes Z (in Fig. 1):

$$K_x = J_e\dot{\alpha}_1 + K_z\beta_1; \; K_y = J_e\dot{\beta}_1 - K_z\alpha_1; \; K_z = J_z u^2\omega. \quad (4)$$

Based on the theorem on the motion of the center of mass of the system and the theorem on the change in kinetic momentum, it was obtained

$$\left.\begin{aligned} m\ddot{x}_c &= -F_{1x} - F_{2x}; \; dK_x/dt = -F_{1y}L_1 + F_{2y}L_2; \\ m\ddot{y}_c &= -F_{1y} - F_{2y}; \; dK_y/dt = -F_{1x}L_1 + F_{2x}L_2. \end{aligned}\right\}. \quad (5)$$

After substituting dependencies (1) – (4) into (5), a system of equations for oscillations of an unbalanced rotor with DVD is obtained:

$$\left.\begin{aligned}
&m\left(L_2\ddot{x}_1+L_1\ddot{x}_2\right)/L+c_1x_1+c_2x_2=me\omega^2\sin\omega t;\\
&\frac{m}{L}\left(L_2\ddot{y}_1+L_1\ddot{y}_2\right)+c_1y_1+c_2y_2=me\omega^2\cos\omega t;\\
&J_e\left(\ddot{y}_1-\ddot{y}_2\right)/L+J_zu^2\omega\left(\dot{x}_2-\dot{x}_1\right)/L+c_1L_1y_1-c_2L_2y_2=\left(J_e+J_zu^2\right)\delta\omega^2\sin\left(\omega t-\varepsilon\right);\\
&J_e\left(\ddot{x}_2-\ddot{x}_1\right)/L-J_zu^2\omega\left(\dot{y}_1-\dot{y}_2\right)/L-c_1L_1x_1+c_2L_2x_2=\left(J_e+J_zu^2\right)\delta\omega^2\cos\left(\omega t-\varepsilon\right)
\end{aligned}\right\}.$$

(6)

Partial solutions of the system of equations (6) have the form:

$$\left.\begin{aligned}
x_1&=A_1\sin\omega t+B_1\cos\omega t;\quad y_1=A_2\cos\omega t+B_2\sin\omega t;\\
x_2&=A_3\sin\omega t+B_3\cos\omega t;\quad y_2=A_4\cos\omega t+B_4\sin\omega t.
\end{aligned}\right\}, \tag{7}$$

As a result of substituting equations (7) into (6), we obtained:

$$\left.\begin{aligned}
&(c_1-mL_2\omega^2/L)A_1+c_2-(mL_1\omega^2/L)A_3=me\omega^2;\\
&(c_1-mL_2\omega^2/L)A_2+(c_2-mL_1\omega^2/L)A_4=me\omega^2;\\
&(c_1L_1-J_e\omega^2/L)A_2-(c_2L_2-J_e\omega^2/L)A_4+J_zu^2\omega^2\left(A_3-A_1\right)/L=-\left(J_e+J_zu^2\right)\delta\omega^2\sin\varepsilon;\\
&-(c_1L_1-J_e\omega^2/L)A_1+(c_2L_2-J_e\omega^2/L)A_3+J_zu^2\omega^2\left(A_2-A_4\right)/L=\left(J_e+J_zu^2\right)\delta\omega^2\sin\varepsilon,
\end{aligned}\right\}$$

(8)

$$\left.\begin{aligned}
&\left(c_1-\frac{mL_2}{L}\omega^2\right)B_1+\left(c_2-\frac{mL_1}{L}\omega^2\right)B_3=0;\\
&\left(c_1-mL_2\omega^2/L\right)B_2+\left(c_2-mL_1\omega^2/L\right)B_4=0;\\
&\left(c_1L_1-J_e\omega^2/L\right)B_2-\left(c_2L_2-J_e\omega^2/L\right)B_4+J_zu^2\omega^2\left(B_3-B_1\right)/L=\left(J_e+J_zu^2\right)\delta\omega^2\cos\varepsilon;\\
&-\left(c_1L_1-J_e\omega^2/L\right)B_1+\left(c_2L_2-J_e\omega^2/L\right)B_3-J_zu^2\omega^2\left(B_2-B_4\right)/L=\left(J_e+J_zu^2\right)\delta\omega^2\cos\varepsilon.
\end{aligned}\right\}.$$

(9)

To simplify the writing of equations (8), (9), the following notation is introduced:

$$a_1=c_1-mL_2\omega^2/L;\ a_2=c_2-mL_1\omega^2/L;\ a_3=c_1L_1-J_e\omega^2/L;$$

$$a_4=c_2L_2-J_e\omega^2/L;\quad a_5=J_zu^2\omega^2/L;\quad b_1=me\omega^2;\quad b_2=\left(J_e+J_zu^2\right)\delta\omega^2.$$

Received:

$$\left.\begin{aligned}&a_1B_1+a_2B_3=0;\\&a_1B_2+a_2B_4=0;\\&a_3B_2-a_4B_4+a_5\left(B_3-B_1\right)=b_2\cos\varepsilon;\\&-a_3B_1+a_4B_3+a_5\left(B_2-B_4\right)=b_2\cos\varepsilon,\end{aligned}\right\},\quad \left.\begin{aligned}&a_1A_1+a_2A_3=b_1;\\&a_1A_2+a_2A_4=b_1;\\&a_3A_2-a_4A_4+a_5\left(A_3-A_1\right)=-b_2\sin\varepsilon;\\&-a_3A_1+a_4A_3+a_5\left(A_2-A_4\right)=b_2\sin\varepsilon.\end{aligned}\right\}. \tag{10}$$

As a result of adding the first two equations of each of the systems (10) and subtracting the last two equations, we obtained

$$\left.\begin{aligned}&\begin{pmatrix}a_1 & a_2\\ \left(a_3-a_5\right) & -\left(a_4-a_5\right)\end{pmatrix}\cdot\begin{pmatrix}A_1+A_2\\ A_3+A_4\end{pmatrix}=\begin{pmatrix}2b_1\\ -2b_2\sin\varepsilon\end{pmatrix};\\&\begin{pmatrix}a_1 & a_2\\ -\left(a_3+a_5\right) & \left(a_4+a_5\right)\end{pmatrix}\cdot\begin{pmatrix}A_1-A_2\\ A_3-A_4\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix},\end{aligned}\right\}, \tag{11}$$

$$\left.\begin{aligned}&\begin{pmatrix}a_1 & a_2\\ \left(a_3-a_5\right) & -\left(a_4-a_5\right)\end{pmatrix}\cdot\begin{pmatrix}B_1+B_2\\ B_3+B_4\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix};\\&\begin{pmatrix}a_1 & a_2\\ -\left(a_3+a_5\right) & \left(a_4+a_5\right)\end{pmatrix}\cdot\begin{pmatrix}B_1-B_2\\ B_3-B_4\end{pmatrix}=\begin{pmatrix}0\\ 2b_2\cos\varepsilon\end{pmatrix}.\end{aligned}\right\} \tag{12}$$

The solutions of the resulting systems have the form:

$$A_1+A_2=\frac{\begin{vmatrix}2b_1 & a_2\\ -2b_2\sin\varepsilon & -\left(a_4-a_5\right)\end{vmatrix}}{\begin{vmatrix}a_1 & a_2\\ \left(a_3-a_5\right) & -\left(a_4-a_5\right)\end{vmatrix}}=\frac{-2b_1\left(a_4-a_5\right)+2a_2b_2\sin\varepsilon}{-a_1\left(a_4-a_5\right)-a_2\left(a_3-a_5\right)};$$

$$A_3+A_4=\frac{\begin{vmatrix}a_1 & 2b_1\\ \left(a_3-a_5\right) & -2b_2\sin\varepsilon\end{vmatrix}}{\begin{vmatrix}a_1 & a_2\\ \left(a_3-a_5\right) & -\left(a_4-a_5\right)\end{vmatrix}}=\frac{-2a_1b_2\sin\varepsilon-2b_1\left(a_3-a_5\right)}{-a_1\left(a_4-a_5\right)-a_2\left(a_3-a_5\right)};$$

$$A_1-A_2=0;\quad A_3-A_4=0.\qquad B_1+B_2=0;\qquad B_3+B_4=0 \quad .$$

$$B_1-B_2=\frac{\begin{vmatrix}0 & a_2\\ 2b_2\cos\varepsilon & \left(a_4+a_5\right)\end{vmatrix}}{\begin{vmatrix}a_1 & a_2\\ -\left(a_3+a_5\right) & \left(a_4+a_5\right)\end{vmatrix}}=\frac{-2a_2b_2\cos\varepsilon}{a_1\left(a_4+a_5\right)+a_2\left(a_3+a_5\right)};$$

$$B_3 - B_4 = \frac{\begin{vmatrix} a_1 & 0 \\ -(a_3 - a_5) & 2b_2\cos\varepsilon \end{vmatrix}}{\begin{vmatrix} a_1 & a_2 \\ -(a_3 + a_5) & (a_4 + a_5) \end{vmatrix}} = \frac{2a_1 b_2 \cos\varepsilon}{a_1(a_4 + a_5) + a_2(a_3 + a_5)} \qquad (13)$$

The last two equations of system (13) have trivial solutions $A_1 = A_2; \quad A_3 = A_4;$ $B_1 = -B_2; \quad B_3 = -B_4$, the denominators correspond to frequency equations , the roots of which determine the critical angular velocities of the oscillatory system:

$$\left.\begin{aligned} f_1(\omega) = a_1(a_4 - a_5) + a_2(a_3 - a_5) = 0; \\ f_2(\omega) = a_1(a_4 + a_5) + a_2(a_3 + a_5) = 0, \end{aligned}\right\}, \qquad (14)$$

Solutions of system (14) have the form

$$\left.\begin{aligned} A_1 = A_2 = \frac{b_1(a_4 - a_5) - a_2 b_2 \sin\varepsilon}{a_1(a_4 - a_5) + a_2(a_3 - a_5)}; \\ A_3 = A_4 = \frac{a_1 b_2 \sin\varepsilon + b_1(a_3 - a_5)}{a_1(a_4 - a_5) + a_2(a_3 - a_5)}, \end{aligned}\right\}, \qquad \left.\begin{aligned} B_1 = -B_2 = \frac{-a_2 b_2 \cos\varepsilon}{a_1(a_4 + a_5) + a_2(a_3 + a_5)}; \\ B_3 = -B_4 = \frac{a_1 b_2 \cos\varepsilon}{a_1(a_4 + a_5) + a_2(a_3 + a_5)}. \end{aligned}\right\}.$$

(15)

Equations of motion (7) are presented in amplitude form

$$\left.\begin{aligned} x_1 = r_A \sin(\omega t - \varphi_A); \quad y_1 = r_A \cos(\omega t - \varphi_A); \\ x_2 = r_B \sin(\omega t - \varphi_B); \quad y_2 = r_B \cos(\omega t - \varphi_B). \end{aligned}\right\}. \qquad (16)$$

где r_A, r_B, φ_A, φ_B – respectively, the elastic deformation modules and vibration phases in supports *A* and *B*.

Equating the right-hand sides of equations (7) and (16), obtained

$$\left.\begin{aligned} r_A = \sqrt{A_1^2 + B_1^2}; \quad r_B = \sqrt{A_3^2 + B_3^2}; \\ \mathrm{tg}\,\varphi_A = -B_1/A_1; \quad \mathrm{tg}\,\varphi_B = -B_3/A_3. \end{aligned}\right\}. \qquad (17)$$

From formulas (17) it follows that the oscillations of an unbalanced rotor with a DVD correspond to its “direct” precession with angular velocity ω. Installed. that the imbalance of the oscillatory system “rotor-DVD” does not caused oscillations associated with “reverse” precession.

The equations of motion of point D of the cutting tool edge are presented as the sum of two harmonic functions of the same frequency:

$$\left.\begin{aligned} x_D &= x_1 + (L_1 + L_3)(x_2 - x_1)/L = \\ &= [1-(L_1+L_3)/L]r_1\sin(\omega t - \varphi_1) + (L_1+L_3)r_2\sin(\omega t - \varphi_2)/L; \\ y_D &= y_1 - (L_1 + L_3)(y_1 - y_2)/L = \\ &= [1-(L_1+L_3)/L]r_1\cos(\omega t - \varphi_1) + (L_1+L_3)r_2\cos(\omega t - \varphi_2)/L. \end{aligned}\right\}$$

Amplitude of rotor oscillations at point D

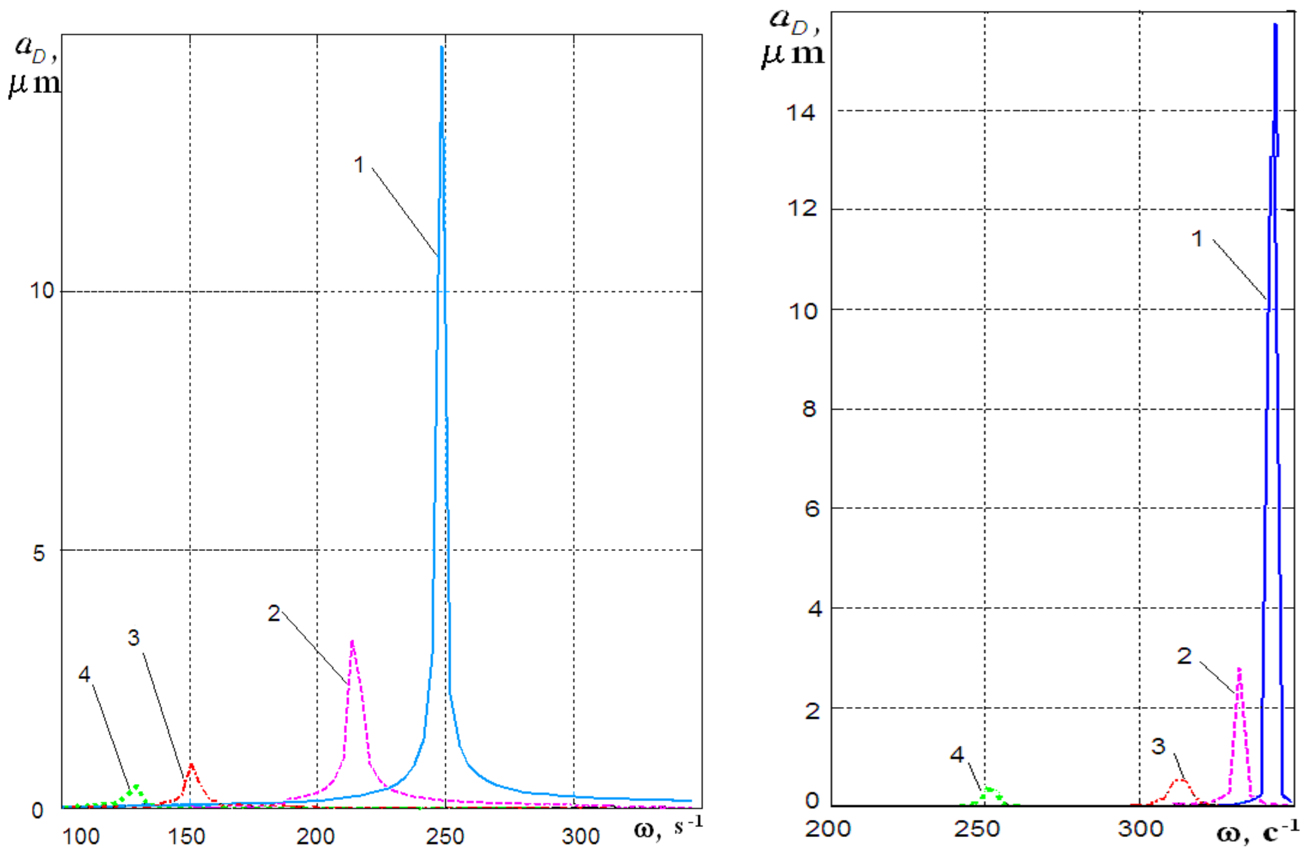
$$a_D = \sqrt{x_D^2 + y_D^2} = \sqrt{(1-\psi)^2 r_1^2 + \xi\psi^2 r_2^2 + 2(1-\psi)\psi r_A r_B \cos(\varphi_B - \varphi_A)},$$

$$\psi = (L_1 + L_3)/L.$$

For numerical modeling of the characteristics of forced vibrations of an unbalanced mandrel with DVD, the following values of the parameters of the calculation model were adopted: L=0,5 m; L_1=0,48 m; L_2=0,02 m; L_3=0,32 m; c_1=c_2=$5\cdot10^6$ N/m; m=5,0 kg; e=0,001…0,01 μm; J_e=0,15 кг·м^2; J_z=0,031 kg·m^2; u=1…10; ω=0…2000 s^{-1}; D_{st}=0,0075…0,075 g·mm; δ=0,001 рад; ε=0,001 rad; F_0=50 N [394, 395].

An analysis of the calculated functions of the oscillation amplitude at point D of a statically unbalanced rotor showed that the critical rotation speed of the rotor without DVD corresponds to a value of 250 s-1, while the amplitude of oscillations of the tool cutting edge reaches 15 μm (in Fig. 3, *a*). As a result of the influence of the DWD at a gear ratio u=10, the amplitude of oscillations at the critical speed of rotation of the mandrel is reduced by more than 20 times. At the same time, in the region of critical frequencies, the amplitudes of rotor oscillations in the sections on the left and right supports practically coincide, and the values of the critical frequencies shift towards reducing the oscillation amplitude by 12% [394, 395].

When the system is statically balanced and the imbalance D_{st} is reduced by 10 times (from 0.075 g·mm to 0.0075 g·mm), a decrease in the amplitude of rotor oscillations by 15 times or more is observed. A further reduction in the amplitude of rotor oscillations is achieved when the DVD is turned on.



a) *b*)

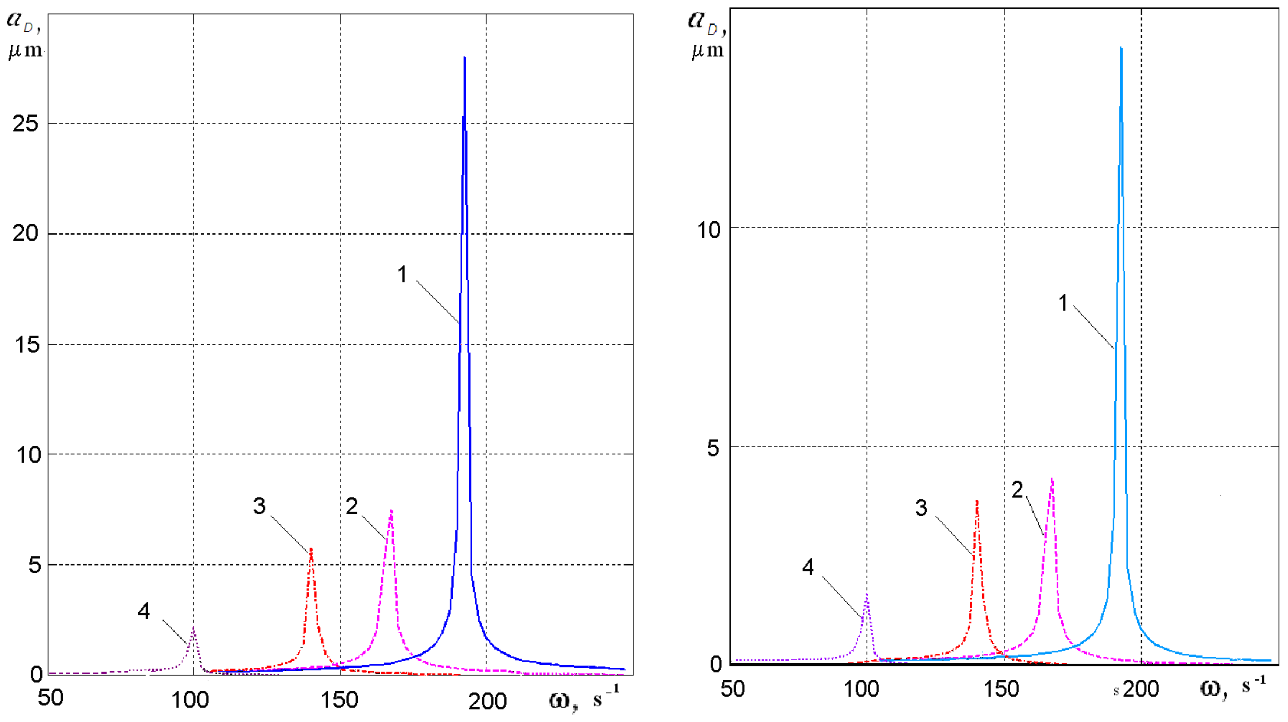
Figure 3. Frequency response graphs for point *D*:

a) D_{st}=0,075 g·mm и δ=0; *b*) D_{st}=0 и δ=0,001 rad
(the numbering of the curves corresponds to: 1 – *u*=1; 2 – *u*=4; 3 – *u*=6; 4 – *u*=10)

To model the amplitude function of elastic vibrations of the rotor in characteristic sections, taking into account dynamic imbalance, the vibration equations of the tool cutting edge (point *D*) are presented in amplitude form [396].

$$\left.\begin{aligned} x_D &= x_1 + (L_1 + L_3)(x_2 - x_1)/L = \\ &= [1 - (L_1 + L_3)/L] r_A \sin(\omega t - \varphi_A) + (L_1 + L_3) r_B \sin(\omega t - \varphi_B)/L; \\ y_D &= y_1 - (L_1 + L_3)(y_1 - y_2)/L = \\ &= [1 - (L_1 + L_3)/L] r_A \cos(\omega t - \varphi_A) + (L_1 + L_3) r_B \cos(\omega t - \varphi_B)/L. \end{aligned}\right\}$$

It has been established that for a dynamically unbalanced mandrel, the use of DVD ensures a reduction in the amplitude of forced vibrations of the tool cutting edge by more than 50 times (in Fig. 3, *b*).



a) *b*)

Figure 4. Frequency response graphs at point *D*:
a) D_{st} =0, δ =0; *b)* D_{st} =0,075 g·mm, δ=0,001 rad
(the numbering of the curves corresponds to: 1 – *u*=1; 2 – *u*=4; 3 – *u*=6; 4 – *u*=10)

As result of the study, the following conclusions were drawn:

1. For a statically unbalanced toolholder, the use of a DVD with a built-in BFM ensures a reduction in the amplitude of oscillations at the critical rotation speed, for example, at *u*=10, by more than 20 times.

2. For a dynamically unbalanced mandrel, the use of a DVD with a built-in BFM ensures a reduction in the amplitude of forced vibrations of the tool cutting edge by more than 50 times [396, 397].

3. Further refinement of the calculated models of forced vibrations of a tool mandrel with an adjustable DVD requires taking into account the influence of dissipation factors in the kinematic bearings of the BFM [398].

4. No less relevant is the study and prediction of the reliability of the DVD, taking into account the heating and wear of the contact surfaces of the ball mill in the limited lubrication mode [399].